Resale of information and stability of networks

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Abstract

This paper explores the effect of strategic incentives on the shape of social and economic networks in the context of information resales. We first determine the relationship between the network structure and agents' relative market power in the resale of information, using the useful concept of *simple networks.* We then provide the characterization of efficient and stable network architecture. We show that a simple network is stable if and only if it consists of one spoke and one circle of size 5. We also show for general networks that stable networks cannot contain strong blocks nor line segments of length > 4. We finally show that stable networks are efficient. We impose no direct costs of maintaining links to emphasize the strategic aspect of networks. Nevertheless, we find that agents do not establish links to all other agents in a stable network. *JEL Classification:* C72; D00; L00.

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1. Introduction

One of the most distinguishing characteristics of information is non-rivalry. While most ordinary goods and services can only be consumed by a limited group of people, information can be transferred to other people without decreasing the consumption of the original possessor. For example, I can share my information about stock prices or good restaurants in town with others, and I still possess the information.[1] In addition, information can be reproduced with virtually no cost.

Due to these characteristics, trades in valuable information often entail resales. That is, once an initial possessor of information sells it to other agents, buyers compete strategically with the seller and other buyers to resell the information. We note that, for many types of information, there are no enforceable means of prohibiting the resale.[2] As specific examples, we take business or technical information, professional newsletters or advertising services listing valuable information.

This paper studies the architecture of efficient and stable networks when agents strategically establish links to obtain a better outcome in the resale of information. The analysis is performed with a two-stage game in which agents first establish links, and then participate in the resale process where an agent can trade the information with another agent if and only if the two are linked.

We start with the analysis of the second stage of resales, and determine the relationship between the network structure and agents' relative market power in the resale of information. We consider a stylized resale process when the link pattern as well as

[1] Of course, it is a different question whether I want to share the information. In other words, the possessor of the information can refuse to share it when she wishes to. Therefore, information can be excludable while it is non-rivalrous.

[2] There simply does not exist any legal protection for many types of information. Even for the types of information that are protected by the intellectual property laws, the enforcement is far from perfect.

the initial possessor of information is given, and find how the total value of information is distributed among agents. We characterize the structure of networks for the resale of information with the useful concept of a *simple network,* which is a connected network where only the initial possessor of information can exert some monopoly power over others.[3] We first determine the distribution of values for simple networks, and then show that every network can be reduced to a simple network.

The main contribution of this paper is the characterization of efficient and stable network architecture. To study the first stage of network formation, we impose a symmetry assumption that (i) every agent has the same valuation for the information, and (ii) each agent becomes the initial possessor of information with equal probability (or alternatively, each agent initially possesses a distinct piece of information.) We first determine the architecture of a simple network that is stable. We prove in Theorem 1 that the only stable simple network architecture is the one consisting of a circle of size 5 and a spoke.[4] We next consider the stability of arbitrary networks. We introduce the concept of a *block,* which is a maximal set of agents with the property that there exist at least two internally disjoint paths from any agent to any other agent. We also define the concept of a *strong block*, which is intuitively a block that is robust to link deletions.[5] We prove in Theorem 2 that a stable network cannot contain strong blocks nor line segments of length > 4.[6] We finally prove in Theorem 3 that every stable network is efficient.

This paper is a contribution to the fast-growing literature on the formation of networks. Papers include Jackson and Wolinsky (1996), Dutta and Mutuswami (1997), Bala and Goyal (2000), and Kranton and Minehart (2001).[7] Jackson (2003) provides a com-

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[3] See the next section for a precise definition.

[4] The architecture is depicted in Figure 2.

[5] See Section 3 for precise definitions.

[6] A possible architecture is depicted in Figure 5.

[7] See also the special issue of *Review of Economic Design* (Vol. 5, No. 2-3, 2000) on the formation of groups. This literature is different from the cooperative game-theoretic approach of coalition formation, as in Myerson (1977) or Slikker (2000), in that the value of a network as well as individuals' payoffs can depend on the exact architecture, i.e., on how individuals are connected.

prehensive survey of this literature. We contribute to the literature by posing a new environment, namely, the resale of information in networks, to analyze agents' strategic incentives to form a network.[8] In particular, unlike most other papers in this area, we impose no direct costs of links to emphasize the strategic aspect of social and economic networks.

The rest of the paper is organized as follows. In the next section, we determine the relationship between the network structure and the value distribution. We provide main results in Section 3, which contains characterizations of efficient and stable network architecture. Section 4 discusses the robustness of main results and directions for future research. Most technical proofs are gathered in Appendix A.

2. Network structure and distribution of values

2.1. Resale of information

Suppose that there is a particular piece of information that can be reproduced costlessly from any copies of the original. Let $N = \{1, \ldots, n\}$ be a set of agents, and let v^i for $i \in N$ be agent i's valuation of the information. We assume that $v^i > 0$ for all $i \in N$. We further assume $n \geq 4$ throughout the paper since otherwise the discussion is rather vacuous.

The information is diffused through a resale process. Assume without loss of generality that agent 1 is the initial monopolistic possessor of the information. Agent 1 first sells the information to other agents, and other agents decide whether to purchase it. Once an agent purchased the information, she can resell it to other non-possessors of information. Therefore, possessors of information compete to (re)sell the information to non-possessors. The resale process continues for finite time periods, after which the consumption stage occurs.

[8] Hence, this paper is a network theory of trades in information, as compared to Kranton and Minehart's (2001) network theory of trades in ordinary goods. Resale of information without network structure was studied in several papers including Muto(1986, 1990) and Nakayama and Quintas (1991).

We impose a network structure on the set of agents in the sense that an agent can purchase the information only from the possessors she is *linked* to. Formally, a network on the set of agents is a $n \times n$ symmetric matrix G such that $g_{ij} = g_{ji} = 1$ if agent i and j are linked and $g_{ij} = g_{ji} = 0$ otherwise. We let $g_{ii} = 1$ for all $i \in N$ as a convention. Therefore, a network structure is represented by a non-directed graph. For any $i \in N$, call $L(i) = \{j \in N \setminus \{i\} : g_{ij} = 1\}$ as the set of i's neighbors. Likewise, for any subset of agents $S \subseteq N$, call $L(S) = \bigcup_{i \in S} L(i) \setminus S = \{j \in N \setminus S : g_{ij} = 1 \text{ for some } i \in S\}$ as the set of S's neighbors.[9] Therefore, when the set of possessors is S, resales can occur only between S and $L(S)$.

2.2. Distribution of values

In this subsection, we study how the total value of the information is distributed among agents. In other words, we determine the final payoff each agent receives from the resale process. We first characterize the distribution of values for a particular kind of networks called *simple networks*, and then generalize to arbitrary networks.

For a given network G and $i, j \in N$, a path q from agent i to agent j is a set of links from i to j. That is, we say that there exists a path q from agent i to agent j if there exist distinct agents $i_1, i_2, \ldots, i_m$ such that $g_{i,i_1} = g_{i_1,i_2} = \cdots = g_{i_m,j} = 1$.[10] The length of a path is the number of its links. As a convention, we treat a link as a path of length 1. We say that the network is connected if there exists a path between any two agents. We first define several important sets.

Definition 1. Consider a connected network G.

(i) The set of i's predecessors in path $q = \{1, i_1, \ldots, i_m, i\}$ from 1 to i is defined as $A(i, q) = \{1, i_1, i_2, \ldots, i_m\}$.

(ii) The set of i's critical predecessors is defined as $A(i) = \bigcap_{q \in Q(i)} A(i, q)$, where $Q(i)$ is

[9] Note that we slightly abuse the notation here since $L(i)$ is in fact $L(\{i\})$.

[10] Since networks are represented by non-directed graphs, we also have $g_{i_1,i} = g_{i_2,i_1} = \cdots = g_{j,i_m} = 1$. We will present only one of the symmetric relations g_{ij} and g_{ji} for convenience.

the set of all paths from 1 to i.

(iii) The set of j's captive descendants is defined as $D(j) = \{i \in N \mid j \in A(i)\}$.

(iv) The set of critical agents is defined as $C = \{j \in N \mid D(j) \neq \emptyset\}$. Each agent in C has some monopoly power over her captive descendants, as we will see below.

We provide examples illustrating these concepts. See also Figure 1.

Example 1. In the complete network where $g_{ij} = 1$ for all i, j in N, we have $A(1) = \emptyset$ and $A(i) = \{1\}$ for all $i \in N \setminus \{1\}$, and $D(1) = N \setminus \{1\}$ and $D(j) = \emptyset$ for $j \in N \setminus \{1\}$. Therefore, $C = \{1\}$. In a line network where $g_{1,2} = g_{2,3} = \ldots = g_{n-1,n} = 1$ and $g_{ij} = 0$ for all other (i, j) pairs with $i \neq j$, we have $A(1) = \emptyset$ and $A(i) = \{1, \ldots, i-1\}$ for $i \in N \setminus \{1\}$, and $D(j) = \{j+1, \ldots, n\}$ for $j \in N \setminus \{n\}$ and $D(n) = \emptyset$. Therefore, $C = \{1, \ldots, n-1\}$.

We now introduce the important concept of *simple networks.* In Proposition 2 below, we show that every connected network can be reduced to a simple network insofar as we are concerned with the distribution of values.

Definition 2. A simple network is a connected network where agent 1 is the only critical agent, i.e., $C = \{1\}$.

Given a simple network, partition the set $N \setminus \{1\}$ by the following equivalence relation: $i \sim j$ if and only if there is a path between i and j not involving agent 1. That is, $i \sim j$ if and only if there is a set of agents $\{i_1, \ldots, i_m\}$ with $1 \notin \{i_1, \ldots, i_m\}$ such that $g_{i,i_1} = g_{i_1,i_2} = \cdots = g_{i_m,j} = 1$. It is clear that this relation is reflexive, symmetric, and transitive.[11] Therefore, $N \setminus \{1\} = N_1 \cup \ldots \cup N_R$ where each N_r is a nonempty subset of agents and $N_r \cap N_{r'} = \emptyset$ for all $r \neq r'; r, r' = 1, \ldots, R$. We can easily see from the definition of the equivalence relation that there is no link between $i \in N_r$ and $j \in N_{r'}$ for all i, j in $N \setminus \{1\}$ and $r \neq r'$. We also have the following proposition. (We will denote the cardinality of a set S by $|S|$ in what follows.)

[11] $i \sim i$ since we let $g_{ii} = 1$ as a convention. Hence, the relation $\sim$ is reflexive.

Proposition 1. *Each N_r is either (i) a one-agent set, or (ii) a set of more than one agent such that, in the subnetwork on $\{1\} \cup N_r$, each agent has at least two links.*

Proof. Consider a set N_r with $|N_r| \geq 2$ and pick any agent i in N_r. Agent $i \in N_r$ has a path to another agent $j \neq i$ in N_r not involving agent 1. Therefore, i has a link to another agent, say k, in N_r. Since i has a path from agent 1 (because the network is connected), if the link to k is the unique link i has then we must have $i \in D(k)$. This, however, is a contradiction since agent 1 is the only critical agent in a simple network.

We next prove that agent 1 has at least two links in the subnetwork on $\{1\} \cup N_r$. Suppose agent 1 has only one link to an agent, say i, in N_r. Then, because the network is connected, i is a critical predecessor for all $j \in N_r \setminus \{i\}$. This, however, is again a contradiction since agent 1 is the only critical agent in a simple network. $\diamondsuit$

Each N_r will be called a *petal.* A petal of the first type in the proposition (that is, a petal with $|N_r| = 1$) will be called a *spoke,* and a petal of the second type will be called a *nontrivial petal.* It is obvious that the only agent in a spoke has a unique path from agent 1 (in fact, has a link with agent 1), and agents in a nontrivial petal have at least two internally disjoint paths from agent 1. Note that two paths are said to be *internally disjoint* if they do not share a common intermediate agent. That is, two paths $q_1 = \{i, i_1, \ldots, i_m, j\}$ and $q_2 = \{i, j_1, \ldots, j_l, j\}$ from agent i to agent j are internally disjoint if $q_1 \cap q_2 = \{i, j\}$.

The distribution of values among agents is generally dependent on the nature of trading institution. For example, if a bargaining institution is employed, then agent 1 may get only a half of the valuation v^i when she sells the information to agent i of a spoke. On the other hand, if agent 1 can make a take-it-or-leave-it offer to agent i, then she can charge the maximum price of v^i. We may also transform the situation into a graph-restricted TU (transferable utilities) game and apply various axiomatic solution concepts.

Although it is certainly an important subject to see how the distribution of values in a given network changes across different trading institutions and/or different solution concepts, we do not delve into this subject since it is beyond the scope of this paper. Instead, we adopt the following specific configuration of value distribution in this paper, and characterize the efficient and stable network architecture in the next section.

For each $r = 1, \dots, R$, let $\pi_r = \max\{v^i | i \in N_r \cap L(1)\}$, and let $H_r = \arg\max\{v^i | i \in N_r \cap L(1)\}$. That is, an agent in H_r is one of those agents who are agent 1's neighbors in N_r and whose valuation for the information is not lower than any other agent in $N_r \cap L(1)$, and π_r is the valuation of agents in H_r. For a spoke $N_r = \{j\}$, it is obvious that $H_r = \{j\}$ and $\pi_r = v^j$. On the other hand, for a nontrivial petal, H_r may contain more than one agent. In this case, we assume that a particular agent h_r is chosen with equal probability among H_r. Let $\{h_r\}_{r=1,\dots,R}$ be one such selection. Then, agent i's final payoff after the resale process in a *simple network* G is given as

$$\phi^i(G) = \begin{cases} v^1 + \sum_{r=1}^{R} \pi_r & \text{if } i = 1, \\ 0 & \text{if } i = h_r \text{ for } r = 1, \dots, R, \\ v^i & \text{otherwise.} \end{cases} \qquad (*)$$

This configuration of value distribution roughly corresponds to the trading institution where sellers have all the bargaining power, but they compete in a Bertrand fashion in the resale process. To be more specific, when agent 1 sells the information to a spoke $N_r = \{j\}$, she charges v^j and extracts all the gains from trade. When agent 1 sells the information to a nontrivial petal, she first sells it to a linked buyer with the maximum valuation, i.e., to h_r, at the price of π_r. After this initial sale, the information is diffused to other agents of this nontrivial petal at zero price since agent 1 and agent h_r now price-compete to resell the information.[12]

We next show that the distribution of values for any connected network can be determined by transforming the network into a simple network.

[12] A specific diffusion game that supports this configuration can be constructed. This construction adapts Muto's (1986) information good game, which formalizes the resale process, to arbitrary networks. Details may be provided upon request.

Proposition 2. *Every connected network can be reduced to a simple network.*

Proof. Fix any connected network G. For each $j \in C = \{j \in N \mid D(j) \neq \emptyset\}$, define the subnetwork starting from j, denoted by $G_s(j)$, as the subnetwork on the set $\{j\} \cup D(j)$. That is, $G_s(j)$ is the submatrix obtained from the original matrix G by keeping only the rows and columns corresponding to the set $\{j\} \cup D(j)$ of agents. We replace $G_s(j)$ by a representative agent j equipped with an imputed valuation $\overline{v}^j$ in a recursive manner. To do this, impose a precedence relation $\prec$ on the set C of critical agents as:

$$j \prec i \text{ if and only if } j \in A(i).$$

Observe that $G_s(j)$ with $j \not\prec j'$ for all $j' \in C$ is a simple network. Therefore, we can compute agent j's payoff in the subnetwork as treating agent j as the initial possessor of the information. We set $\overline{v}^j$ to be equal to this payoff and replace $G_s(j)$ with a representative agent j equipped with a valuation $\overline{v}^j$. Note that we can contract $G_s(j)$ into j without losing any information about the network structure outside the subnetwork $G_s(j)$ since there is no link between any agent in $D(j)$ and another agent in $N \setminus (\{j\} \cup D(j))$.[13] Do the process recursively along the precedence relation $\prec$ for all $j \in C$ except agent 1. For each agent $i \in N \setminus \cup_{j \in C}(\{j\} \cup D(j))$, assign $\overline{v}^i = v^i$. Then we are given a simple network with agent 1 being the only critical agent and other agents equipped with valuations $\overline{v}^i$'s, which we have just derived. $\diamondsuit$

Note that we determine the distribution of values by recursively applying $(*)$ along the precedence relation, starting from the outermost subnetworks which are by construction simple. Therefore, we can extend the domain of the function $\phi^i(\cdot)$ to all connected networks.[14] Since any network can be decomposed into connected components, we can in fact determine the distribution of values for an arbitrary network by applying Proposition

[13] Suppose there exist $i \in D(j)$ and $k \in N \setminus (\{j\} \cup D(j))$ such that $g_{ik} = 1$. Since $k \notin D(j)$, there is a path from 1 to k not involving j. Then, there is a path from 1 to i not involving j, contradicting the fact that $i \in D(j)$.

[14] Note, however, that the configuration $(*)$ can only be applied to simple networks. Hence, a given connected network needs to be recursively reduced to a simple network, which is the main point of Proposition 2.

2 to each connected component. That is, the domain of $\phi^i(\cdot)$ can be extended to arbitrary networks. In the remainder of this section, we show how the total value of the information is distributed among agents for some specific networks. Figure 1 depicts the networks.

(a) The complete network: The complete network is a simple network with a unique nontrivial petal and no spoke. Let j be an agent with $v^j = \max\{v^2, \ldots, v^n\}$. Then, by the configuration $(*)$ above, we have $\phi^1(G) = v^1 + v^j$, $\phi^j(G) = 0$, and $\phi^k(G) = v^k$ for all $k \neq 1, j$.

(b) A star network: A star network is a network where there is a central agent $i \in N$ such that $g_{ij} = 1$ for all $j \in N \setminus \{i\}$ and there exists no other link. Suppose that agent 1 is the central agent. Then, the network is a simple network consisting of only spokes. We thus have $\phi^1(G) = \sum_{i=1}^n v^i$ and $\phi^j(G) = 0$ for all $j \neq 1$ by the configuration $(*)$.

(c) A circle network: A circle network is a network where agents are arranged as $\{i_1, \ldots, i_n\}$ with $g_{i_1,i_2} = \ldots = g_{i_{n-1},i_n} = g_{i_n,i_1} = 1$ and there exists no other link. Let j be an agent in $L(1)$ with $v^j = \max\{v^k | k \in L(1)\}$. Since the circle network is a simple network with a unique nontrivial petal and no spoke, we have $\phi^1(G) = v^1 + v^j$, $\phi^j(G) = 0$, and $\phi^k(G) = v^k$ for all $k \neq 1, j$ by the configuration $(*)$.

(d) A line network: A line network is a network where agents are arranged as $\{i_1, \ldots, i_n\}$ with $g_{i_1,i_2} = \ldots = g_{i_{n-1},i_n} = 1$ and there exists no other link. Suppose that $i_1 = 1$. The network is not a simple network. Therefore, we can recursively reduce G beginning with i_{n-1} as in Proposition 2 to conclude that $\phi^1(G) = \sum_{i=1}^n v^i$ and $\phi^j(G) = 0$ for all $j \neq 1$.

[Figure 1 inserted around here]

3. Efficiency and stability of networks

In this section, we analyze the first stage of network formation, where agents strategically form or sever links to enhance their relative market power in the ensuing stage of resales studied above. We introduce a symmetry assumption.

Assumption 1. (Symmetry) (i) every agent has the same valuation v for the information, and (ii) each agent becomes the initial possessor of the information with equal probability, $1/n$.

With Assumption 1, agent i's payoff in the previous section, $\phi^i(G)$, is in fact i's payoff in the network G when agent 1 is endowed with the information initially. From now on, we will denote this payoff by $\phi_1^i(G)$. Generally, agent i's payoff in the network G when agent j is the initial possessor is denoted by $\phi_j^i(G)$. Then, agent i's expected payoff given the network structure G is

$$\Phi^i(G) = \frac{\phi_1^i(G) + \cdots + \phi_n^i(G)}{n}.$$

We note that Assumption 1(ii) can be alternatively formulated as 'each agent initially possesses a distinct piece of information whose value is v for all agents.' In this formulation, $\Phi^i(G)$ is agent i's payoff normalized by the size of network, i.e., n. We now state the notions of efficiency and stability.

Definition 3. A network G is efficient if $\sum_{i \in N} \Phi^i(G) \geq \sum_{i \in N} \Phi^i(G')$ for all other networks G'.

Note that this notion of efficiency is termed as strong efficiency in Jackson and Wolinsky (1996), which is stronger than Pareto efficiency. The notion of stability we use is pairwise stability. As mentioned in Jackson and Wolinsky (1996), pairwise stability is a relatively weak notion among those which account for network formation. As will be shown below, however, this weak notion is strong enough in our framework since it essentially excludes all but one network architecture. In the definition, $G + ij$ denotes the network where a new link ij is added to the original network G, and $G - ij$ denotes the network where the existing link ij is deleted from the original network G.

Definition 4. A network G is stable if (i) for all i and j with $g_{ij} = 1$, $\Phi^i(G) > \Phi^i(G - ij)$ and $\Phi^j(G) > \Phi^j(G - ij)$, and (ii) for all i and j with $g_{ij} = 0$, if $\Phi^i(G + ij) > \Phi^i(G)$ then $\Phi^j(G + ij) \leq \Phi^j(G)$.

Hence, when an agent is indifferent between maintaining a link and not doing so, she chooses not to maintain. Note that this notion of pairwise stability is slightly different from that in Jackson and Wolinsky (1996), which postulates that an agent chooses to maintain a link when she is indifferent.[15]

Let us introduce at this point several terms and notations used hereafter. Consider a given network G over the set $N = \{1, \ldots, n\}$ of agents. We first observe that, depending on the identity of the initial possessor, G may become a simple network or not. For example, a star network with agent 1 being the center becomes a simple network when agent 1 is initially endowed with the information, while it is not a simple network when an agent other than agent 1 is initially endowed with the information. As a convention, we will continue to call a network G a simple network if G turns out to be a simple network when agent 1 is initially endowed with the information. Next, we will denote a simple network as $G = [G_1, \ldots, G_R]$, where G_r is the subnetwork on $\{1\} \cup N_r$. We will say that a petal N_r *constitutes* a certain type of network if G_r is that type of network. For example, we will say that N_r constitutes a circle network if G_r is a circle network. Needless to say, G cannot be represented in this way when agent $i \neq 1$ is the initial possessor. Finally, we will use n_r in place of $|N_r|$ to denote the cardinality of N_r.

In the following, we first characterize the stability of simple networks, and then discuss efficiency and stability of general networks. We remind the reader that we have assumed $n \geq 4$. (When $n = 2$ or $n = 3$, it is trivial to see that a line network connecting every agent is both efficient and stable.)

3.1. Stability of simple networks

We now characterize the structure of stable simple networks (that is, simple networks that are stable) in Propositions 3-8 below, which culminates in Theorem 1. All the proofs of propositions, as well as additional lemmas, can be found in Appendix A.

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[15] The notion of stability we employ can be interpreted in such a way that it implicitly assumes an infinitesimal cost of maintaining a link. See Section 4 for a detailed discussion.

Proposition 3. *If a simple network $G = [G_1, \ldots, G_R]$ is stable, then each of its nontrivial petals constitutes a circle network.*

Proposition 3 implies in particular that the complete network is not stable. The intuition for this proposition is as follows. When an agent establishes an additional link, her probability of being linked to the initial owner of information increases, and so the probability of getting a zero payoff. Therefore, agents try to maintain as few links as possible.

Proposition 4. *If a simple network $G = [G_1, \ldots, G_R]$ is stable, then it contains more than one petal.*

Note that Proposition 4 in fact establishes that circle networks are not stable.

Proposition 5. *If a simple network $G = [G_1, \ldots, G_R]$ is stable, then each of its nontrivial petals has at least three agents. Moreover, if $R = 2$ then each of its nontrivial petals has at least four agents.*

Proposition 6. *If a simple network $G = [G_1, \ldots, G_R]$ is stable, then it contains exactly one spoke.*

Proposition 6 implies that star networks are not stable. The intuition for this proposition is as follows. First, agents do not want to be spokes since this would leave them with zero payoff when agent 1 is the initial possessor. Hence, if the network contains two or more spokes, then agents in spokes have an incentive to establish links. Second, if the network does not contain a spoke (but two or more nontrivial petals by Proposition 4), then agent 1 has an incentive to disconnect links to increase her monopoly power over petals.

Proposition 7. *If a simple network $G = [G_1, \ldots, G_R]$ is stable, then it contains exactly one nontrivial petal.*

We have established the fact that if a simple network is stable, then it contains exactly one spoke and exactly one nontrivial petal which constitutes a circle network. Our final characterization for a stable simple network is:

Proposition 8. *If a simple network is stable, then its nontrivial petal has exactly four agents.*

We will call the circle network made up of the nontrivial petal and agent 1 as the circle part of the simple network G. From Proposition 8, we know that the circle is of size 5. That is, it has 5 agents - agents in the nontrivial petal and agent 1. We have established necessary conditions for a stable simple network through Propositions 3-8. These conditions are in fact sufficient. We thus have the following complete characterization of a stable simple network. See also Figure 2 for a graphical representation.

Theorem 1. *A simple network G is stable if and only if it contains exactly one spoke and exactly one nontrivial petal which constitutes a circle network and has exactly 4 agents. In other words, a stable simple network consists of one spoke and one circle of size 5.*

Proof. We leave it to the reader to show the sufficiency part since it is a straightforward, albeit rather lengthy, exercise. $\diamondsuit$

We want to emphasize that simple networks cover many interesting networks. The complete network, circles, and stars are simple networks. All the networks that have more links than a circle are also simple. One can also add additional spokes to a simple network. Simple networks clearly cover more than just these networks. Moreover, since agents can be renamed, it is sufficient to find the network to be simple with any agent being the initial possessor. Theorem 1 establishes that all the simple networks are not stable except for one architecture.

[Figure 2 inserted around here]

3.2. Efficiency and stability of general networks

We now turn to the discussion of general networks.[16] We define the concept of *blocks* for a given network G.

Definition 5. A block is a subset M of agents such that (i) for any $i, j \in M$, there exist at least two internally disjoint paths from i to j,[17] and (ii) there does not exist a superset of M which satisfies property (i), i.e., the set M is maximal with respect to property (i).[18]

Note that a block contains at least 3 agents. It is a well-known fact in graph theory that two blocks in a network share at most one agent. In addition, an agent in a network who does not belong to a block does not belong to any cycle. That is, she is a *cut-vertex* in that the network has more components without her. Therefore, an arbitrary network is decomposed into blocks and isolated agents who do not belong to any cycle.

We first discuss the stable architecture of blocks. Given an arbitrary network G, consider the subnetwork G_M on a block M. One of the most important features of blocks for our purpose is that the only critical agent in this subnetwork is the initial possessor, no matter who the initial possessor is in M. Suppose $i \in M$ is the initial possessor. By our configuration $(*)$ in Section 2, the information good will be diffused to M at zero price except for one of i's neighbors $L(i)$. On the other hand, agent i will sell the good to a neighbor who has the highest (imputed) valuation. Recall from the proof of Proposition 2 that the imputed valuations of agents in M of the network G, if G is not simple, can be derived by recursively reducing the connected component containing M to a simple network with agent i being the initial possessor.

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[16] I thank an anonymous referee for correcting severe errors in this subsection.

[17] Recall from the discussion after Proposition 1 that two paths are internally disjoint if they do not share a common intermediate agent.

[18] We want to note that this definition of blocks is slightly different from the conventional one. In graph theory, a block is defined to be a subnetwork that satisfies (i) and (ii). We define a block as a set of nodes instead. This is useful for our purpose since nodes are the main units of analysis in our study of strategic stability.

Does agent i, the initial possessor in M, has an incentive to disconnect a link to M? To answer this question, we classify blocks as follows.

Definition 6. Consider a block M and the subnetwork G_M on it.

(i) M is a weak block if M ceases to be a block whenever any one link in G_M is deleted.

(ii) M is a semi-strong block if there exists a link ik in G_M such that M continues to be a block in the new network $G_M - ik$.

(iii) M is a strong block if M continues to be a block in the new network $G_M - ik$ for any link ik in G_M.

A block that constitutes a circle network, which is a block with the minimum number of links, is a weak block. There are other weak blocks, as the network in Figure 3(a) shows. The network in (b) shows a semi-strong but not a strong block, while the one in (c) shows a strong block. Note that the set N of agents in a complete network with $n \geq 4$ is a strong block. Note also that M is a strong block if and only if there exist at least three internally disjoint paths from any agent to any other agent.

[Figure 3 inserted around here]

The answer to the stability of blocks with respect to link deletion depends on whether M continues to be a block in the new network. Let h be an agent in $M \cap L(i)$ with the lowest (imputed) valuation. That is, h is an agent in the set

$$\arg\min\{\overline{v}^j \mid j \in M \cap L(i)\}.$$

Let $G' = G - ih$, and assume that M remains as a block in G'. Then, we have $\phi_i^i(G) = \phi_i^i(G')$. In addition, it is clear that $\phi_j^i(G) = \phi_j^i(G')$ for all $j \notin M$. It is also clear that $\phi_j^i(G) = \phi_j^i(G')$ for all $j \in M \setminus \{i, h\}$ and $\phi_h^i(G) \leq \phi_h^i(G')$. Therefore, since $\Phi^i(G) \leq \Phi^i(G')$, agent i has an incentive to disconnect one of its links in M by Definition 4(i) of stability. The reason is that her probability of being linked to the initial owner of information decreases, and so the probability of getting a zero payoff. This implies that strong blocks cannot be stable.

Matters become complicated when M is not a strong block. Consider the following figure.

[Figure 4 inserted around here]

The set $M = \{1, 2, 3, 4\}$ is a semi-strong block in the network G shown in Figure 4(a). Suppose agent 1 is the initial possessor of the information. When she sells the good to agent 2 or 4, the price she can charge is v. On the other hand, she can charge $2v$ to agent 3. Hence, agent 1 sells the good to agent 3 among M, and h as defined above is either 2 or 4. Let $h = 2$, and consider the situation when 1 disconnects the link 12. In the resulting network $G' = G - 12$, the set $M = \{1, 2, 3, 4\}$ ceases to be a block. In this case, we see that $\phi_1^1(G) = 4 < 5 = \phi_1^1(G')$ and that $\phi_2^1(G) = 1 > 0 = \phi_2^1(G')$. Observe that things get more involved in networks such as the one in Figure 4(b). These simple examples demonstrate that it seems almost impossible to completely characterize the stable network architecture. To summarize the discussion, we have the following proposition.

Proposition 9. *If a network G is stable, then none of its blocks can be a strong block.*

Recall that the circle is the only stable block architecture in simple networks (Proposition 3). By contrast, we have a weaker characterization for general networks. The reason is that we cannot delete an arbitrary link of an agent in general networks since her neighbors may have different imputed valuations.

We now turn to the discussion of isolated agents. Note that the agents not in a block form the tree part of the network, and the tree part is composed of line segments. We ask the length of a line segment in a stable network. There are two types of line segments to consider, closed line segments and open line segments. A *closed line segment* is a line subnetwork both of whose ends are connected to another part of the network, while an *open line segment* is a line subnetwork only one of whose ends is connected to another part of the network. To answer the question, consider a line network of m agents such that $g_{1,2} = g_{2,3} = \cdots = g_{m-1,m} = 1$ and let agents' valuations for the information to be $v^2 = v^3 = \cdots = v^{m-1} = v$ and $v^1, v^m > v$. Observe that a closed line segment is represented by this type of line network with $v^1 \geq 2v$ and $v^m \geq 2v$, and an open line segment is represented by this type of line network with $v^1 \geq 2v$ and $v^m = v$. Regardless of whether a line segment is closed or open, we have:

Proposition 10. *If a network G is stable, then all of its line segments are of length ≤ 4.*

It is an easy exercise to show that an open line segment of length ≤ 4 is stable (in itself). For a closed line segment of size ≤ 4, we cannot determine the stability definitely.

Example 2. Consider a closed line segment with length 3. In this network G, we have $\Phi^i(G) = \frac{1}{3}[v^1 + v + v^3]$. Now, if agents 1 and 3 establish a link, then in the new network $G' = G + 13$, we have

$\Phi^1(G') = \frac{1}{3}[v^1 + v^3 + v^1]$ and $\Phi^3(G') = \frac{1}{3}[v^1 + v^3]$ if $v^1 < v^3$,

$\Phi^1(G') = \frac{1}{3}[v^1 + v^3]$ and $\Phi^3(G') = \frac{1}{3}[v^1 + v^3 + v^3]$ if $v^1 > v^3$, and

$\Phi^1(G') = \frac{1}{3}[v^1 + v^3 + v^1/2]$ and $\Phi^3(G') = \frac{1}{3}[v^1 + v^3 + v^1/2]$ if $v^1 = v^3$.

Therefore, if $v^1 = v^3 > 2v$, then 1 and 3 have an incentive to establish a link, rendering the closed line segment unstable. In all other cases, the line segment is stable. For a closed line segment with length 4, we can find more cases of instability.

Summarizing the discussion, we have the following characterization of stable networks.

Theorem 2. *If a network is stable, then it cannot contain strong blocks nor line segments of length > 4.*

We show a typical shape of stable networks in Figure 5. To complete the picture, we now prove the following proposition for the stand-alone line networks. Note that line networks are not simple networks. This proposition implies in particular that, when the number of agents is less than or equal to 5, line networks are the only stable network architecture.

[Figure 5 inserted around here]

Proposition 11. *Line networks are stable if and only if its length is less than or equal to 5.*

Our final result is on the efficiency of networks.

Theorem 3. *Stable networks are efficient.*

Proof. We first observe that a network is efficient if and only if it is connected since the maximum total value of the information for the set $N = \{1, 2, \ldots, n\}$ of agents, which is equal to nv, will be distributed in some way among n agents if and only if the network is connected. Therefore, all we need to show is that stable networks are connected. Suppose to the effect of contradiction that there exist at least two connected components in a stable network G. Since G is stable, each of its components is either a line network of less than or equal to 5 players or a network made up of blocks and line segments. It is not hard to see that an agent at the end of a line segment has an incentive to establish a link with another agent in other components. It is also easy to see that an agent at the juncture (of a block and a line segment or of blocks) of a network made up of blocks and line segments has an incentive to establish a link with another agent in other components.[19] Since there are at least two such agents whenever a network is not connected, we proved the claim. $\diamondsuit$

Note: The converse of the theorem is obviously not true.

4. Discussion

As mentioned in the Introduction, we have not imposed direct costs of maintaining a link to concentrate on agents' strategic incentives. We note that, even when agents incur strictly positive (direct) costs of links, the analysis in the previous section is not altered as long as the costs are relatively low compared to the valuation v. The reason is that, even without direct costs, agents minimize the number of links due to the pricing of the

[19] Note that a stable network cannot contain a stand-alone circle, as shown in Proposition 4.

information. To put it differently, the value of every link in a stable network is strictly positive to the agents who maintain it.[20] Therefore, the assumption of no direct costs of links is not essential to the analysis.

We have postulated in the definition of stability that when an agent is indifferent between maintaining a link and not doing so, she chooses not to maintain. This is a natural assumption; and especially so when there exist strictly positive costs of links.[21] Nevertheless, we want to briefly discuss the alternative case when an agent chooses to maintain a link when she is indifferent.[22] It is easy to see that a stable network is overconnected in this case compared to the previous analysis. For example, in a network shown in Figure 2, agents 2 and 5 need to maintain an additional link for this to be stable. In addition, line networks for $n \leq 5$ cease to be stable but instead circle networks are stable.[23] We note, however, that the network will not be overly connected. This is easily seen from the above examples. More generally, we observe that the *endogenous* cost of a link to agent i when she establishes a new link with agent j is roughly $\frac{1}{n} \cdot \frac{v}{|L(j)|}$ for most $j \in N$. Therefore, although agents maintain more links in this alternative case, there exist obvious limits due to the endogenous costs of pricing.

We have discovered the efficient and stable network architecture for the resale of information. We have completely characterized the architecture of a stable simple network: It consists of a spoke and a circle of 5 agents. More generally, stable networks cannot contain strong blocks nor line segments of length > 4. We also found that stable networks are efficient. These strong results are obtained under the configuration $(*)$ of value distribution in Section 2. Recall that the configuration $(*)$ corresponds to the trading institution

[20] We leave it to the reader to confirm this argument since it is a straightforward exercise to determine the value of links in any given network.

[21] If we implicitly assume small costs, however, then the efficiency result of Theorem 3 should be reinterpreted as an approximate efficiency result, in the sense that stable networks generate a value arbitrarily close to the efficient value as we choose these costs infinitesimally small.

[22] Note that this is the notion of pairwise stability in Jackson and Wolinsky (1996).

[23] This change in the notion of pairwise stability affects the analysis only via the first part of Definition 4. It has a bite only in five specific lines in the proofs of Propositions 3, 4, 5, 8, and 9. We have indicated them in the proofs by explicitly referring Definition 4(i).

where sellers have all the bargaining power, but they compete in a Bertrand fashion in the resale process. It certainly is worthwhile to see whether the current architecture of stable networks will survive under different configurations, i.e., under different trading institutions. We conjecture that, although the exact shape may be altered, the basic feature of stable networks that they do not have strong blocks of agents will continue to hold for most reasonable trading institutions.[24] Other possible generalizations include asymmetry of agents' valuations as well as asymmetry of agents themselves such that only a subset of agents may ever become initial possessors of the information.

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I thank anonymous referees for many helpful comments and suggestions.

Appendix A

We start by presenting simple observations in the following lemma. We do not provide the proofs since they are straightforward.

Lemma 1. *Consider a simple network* $G = [G_1, \ldots, G_R]$. *We have*

(i) $\phi_1^1(G) = (R+1)v$.

(ii) *For* $i \in N_r$, *we have* $\phi_i^i(G) = \phi_1^1(G)$ *if* $i \in L(1)$, *and* $\phi_i^i(G) = 2v$ *otherwise.*

(iii) *For* $i \in N_r$, *we have*

$$\phi_1^i(G) = \begin{cases} (1 - 1/|N_r \cap L(1)|)v & \text{if } i \in L(1), \text{ and} \\ v & \text{otherwise.} \end{cases}$$

(iv) *For* $i \in N_r$ *and* $j \notin N_r$, *we have* $\phi_j^i(G) = \phi_1^i(G)$.

(v) *When* $R > 1$, *we have* $\phi_j^1(G) = 0$ *for all* $j \in L(1)$ *and* $\phi_j^1(G) = Rv$ *for all* $j \notin L(1)$.

(vi) *When* $R > 1$, *we have* $\phi_j^i(G) = v$ *for all* $i, j \in N_r$ *with* $i \neq j \in L(1)$.

[24] For a very brief example, consider an alternative configuration for four agents such that the initial possessor receives a fraction α of total value, the first purchaser receives a fraction β of total value, and each of the remaining agents receives a fraction of γ of total value. (So, we have $\alpha + \beta + 2\gamma = 1$.) We can easily show that the complete network is not stable as long as $\beta \leq \gamma$: Agent 1 has an incentive to disconnect the link 12 since that will result in $\Phi^1 = v + v(\gamma - \beta)/3$, while her payoff in the complete network is v. Hence, stable networks are incomplete. It is also obvious that stable networks are connected and so efficient.

Note: We have $\phi_1^i(G) = (1 - 1/|N_r \cap L(1)|)v$ if $i \in L(1)$ in case (iii) since the information will be sold to one of the agents in $N_r \cap L(1)$ with equal probability, given that they have the same valuation.

An example would be helpful in understanding these observations.

Example 3. Consider the simple network shown in Figure 2. Then,

(i) $\phi_1^1(G) = 3v$.

(ii) $\phi_2^2(G) = \phi_5^5(G) = \phi_6^6(G) = 3v$, while $\phi_3^3(G) = \phi_4^4(G) = 2v$.

(iii) $\phi_1^2(G) = \phi_1^5(G) = v/2$, $\phi_1^3(G) = \phi_1^4(G) = v$, and $\phi_1^6(G) = 0$.

(iv) $\phi_6^i(G) = \phi_1^i(G)$ for $i = 2, 3, 4, 5$ and $\phi_j^6(G) = \phi_1^6(G) = 0$ for $j = 2, 3, 4, 5$.

(v) $\phi_2^1(G) = \phi_5^1(G) = \phi_6^1(G) = 0$ and $\phi_3^1(G) = \phi_4^1(G) = 2v$.

(vi) $\phi_2^3(G) = \phi_5^3(G) = \phi_2^4(G) = \phi_5^4(G) = v$.

We next have

Lemma 2. *(Decomposition Lemma) Consider a simple network $G = [G_1, \ldots, G_R]$ and suppose the subnetwork $G_{r'}$ changes to $G'_{r'}$. If $G' = [G_1, \ldots, G_{r'-1}, G'_{r'}, G_{r'+1}, \ldots, G_R]$ is a simple network, then $\Phi^i(G) = \Phi^i(G')$ for all $i \in N_r$, $r \neq r'$.*

Proof. We first note that $\phi_1^1(G) = \phi_1^1(G')$ by Lemma 1(i). For $i \in N_r$ with $r \neq r'$, we observe that $\phi_1^i(G) = \phi_1^i(G')$ and $\phi_i^i(G) = \phi_i^i(G')$ by Lemma 1(ii)-(iii). It is also obvious that $\phi_j^i(G) = \phi_j^i(G')$ for all $j \in N_r$. Since $\Phi^i(G) = \frac{1}{n}\sum_{j \in N_r} \phi_j^i(G) + \frac{1}{n}\sum_{j \notin N_r} \phi_j^i(G) = \frac{1}{n}\sum_{j \in N_r} \phi_j^i(G) + \frac{n-n_r}{n}\phi_1^i(G)$ by Lemma 1(iv), we have $\Phi^i(G) = \Phi^i(G')$ for all $i \in N_r$, $r \neq r'$. ♢

Lemma 2 shows that an agent's payoff is not affected by a change in network structure occurring outside of her own petal. This lemma, therefore, implies that agents need not know the behavior outside their own petals. This significantly reduces agents' observational burden. In other words, it is sufficient for each agent to observe only the local behavior in her own petal.

Proof of Proposition 3. Consider a nontrivial petal N_r. We first show that $\{1\} \cup N_r$ is a block, whose concept is introduced in Definition 5 of the text.

Lemma 3. *If N_r is a nontrivial petal in a simple network, then $\{1\} \cup N_r$ is a block.*

Proof. We will show that, for any $i, j \in \{1\} \cup N_r$, there is a cycle through both agents.

Case 1: $i = 1$ or $j = 1$.

Without loss of generality, assume that $i = 1$. Since agent 1 is the only critical agent, there exist two internally disjoint paths from 1 to j. We thus get a cycle concatenating these two paths.

Case 2: $i \neq 1$ and $j \neq 1$.

By definition of N_r, there is a path from i to j not involving agent 1. Moreover, since 1 is the only critical agent, there is a path from 1 to i not involving j, and a path from 1 to j not involving i. We thus get a cycle by concatenating these paths.

It is obvious that $\{1\} \cup N_r$ is maximal with respect to this property. Hence, $\{1\} \cup N_r$ is a block. $\diamondsuit$

We now establish an important fact for weak blocks, whose concept is introduced in Definition 6 of the text. Consider a weak block M, and assume that the (sub)network G_M defined on it is not a circle. Then, there exists an agent i in M with at least 3 links in G_M. Since M is a weak block, it ceases to be a block in the network $G_M - ik$ for any $k \in M \cap L(i)$. On the other hand, since M was originally a block and thus there exist at least two internally disjoint paths between i and k in G_M, agent k is connected to i even in $G_M - ik$. In fact, by the ear decomposition theorem, k has a path, say P, to a cycle, say C, which contains agent i and two of i's links.[25] We claim that the terminus of P, say

[25] The ear decomposition theorem, due originally to Whitney, is presented in most graph theory textbook. See, for example, West (2001). An ear of a graph G is a path in G that is contained in a cycle and is maximal with respect to internal nodes having degree 2. An ear decomposition of G is a decomposition $P_0, \ldots, P_k$ such that P_0 is a cycle and P_i for $i \geq 1$ is an ear of $P_0 \cup \cdots \cup P_i$. The ear decomposition theorem states that a graph has at least two internally disjoint paths for any pair of nodes if and only if it has an ear decomposition. Furthermore, every cycle in such a graph is the initial cycle in some ear decomposition.

agent l, that meets the cycle C is not a neighbor of i, i.e., $l \notin L(i)$.

Suppose $l \in L(i)$. Then, M remains as a block in the network $G_M - il$, contradicting the fact that M is a weak block. Observe that M remains as a block in $G_M - il$ since, in the original network G_M, the path $P + ik$ was an ear of C in a ear decomposition, and hence the concatenation of $P + ik$ with $C - il$ is now becomes the initial cycle in an ear decomposition of $G_M - il$. To summarize the discussion, we have the following lemma.

Lemma 4. *Let M be a weak block, and assume that the subnetwork G_M on this block is not a circle. For any agent $i \in M$ with at least 3 links and for any $k \in M \cap L(i)$, the terminus of the path from k to a cycle containing agent i and two of its links is not a neighbor of i.*

We are ready to prove Proposition 3. Suppose that a nontrivial petal N_r does not constitute a circle network. We will first give a proof for a nontrivial petal N_r such that $\{1\} \cup N_r$ is not a weak block. Then, there exists a link ik in G_r (that is, there exist agents i and k in $\{1\} \cup N_r$ with $g_{ik} = 1$) such that $G'_r = G_r - ik$ is a nontrivial petal and hence $G' = [G_1, \ldots, G_{r-1}, G'_r, G_{r+1}, \ldots, G_R]$ remains a simple network.
Case 1: $i = 1$ or $k = 1$.

Without loss of generality, assume that $i = 1$. We have $\phi_1^1(G) = \phi_1^1(G')$ by Lemma 1(i) and, in addition, $\phi_j^1(G) = \phi_j^1(G')$ for $j \notin N_r$ obviously. Therefore, $\sum_{j \notin N_r} \phi_j^1(G) = \sum_{j \notin N_r} \phi_j^1(G')$. Now, for $j \in N_r$, we consider two subcases.
Subcase 1.1: When $R = 1$. In this case, $\phi_j^1(G)$ for $j \in N_r$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(1)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r} \phi_j^1(G) = n_r v - \sum_{j \in N_r \cap L(1)} v/|L(j)|$. On the other hand, $\sum_{j \in N_r} \phi_j^1(G') = n_r v - \sum_{j \in N_r \cap L(1)} v/|L(j)| + v/|L(k)|$. Therefore, $\Phi^1(G) < \Phi^1(G')$ and agent 1 has an incentive to disconnect the link.
Subcase 1.2: When $R > 1$. In this case, $\phi_j^1(G)$ for $j \in N_r$ is equal to 0 if $j \in L(1)$ and is equal to Rv otherwise. Therefore, $\sum_{j \in N_r} \phi_j^1(G) = (n_r - |N_r \cap L(1)|)Rv$. On the other hand, $\sum_{j \in N_r} \phi_j^1(G') = (n_r - |N_r \cap L(1)|)Rv + Rv$. Therefore, $\Phi^1(G) < \Phi^1(G')$ and agent 1 has an incentive to disconnect the link.

Case 2: $i \neq 1$ and $k \neq 1$.

We will show that agent i has an incentive to disconnect the link. First observe that $g_{1i} = 1$ if and only $g'_{1i} = 1$. Therefore, we have $\phi_i^i(G) = \phi_i^i(G')$ by Lemma 1(i)-(ii). We also have $\phi_j^i(G) = \phi_j^i(G')$ for $j \notin N_r$ by Lemma 1(iii)-(iv). Therefore, $\sum_{j \notin N_r \setminus \{i\}} \phi_j^i(G) = \sum_{j \notin N_r \setminus \{i\}} \phi_j^i(G')$. Now, for $j \in N_r \setminus \{i\}$, we consider two subcases.

Subcase 2.1: When $R = 1$. In this case, $\phi_j^i(G)$ for $j \in N_r \setminus \{i\}$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(i)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) = (n_r - 1)v - \sum_{j \in N_r \cap L(i)} v/|L(j)|$. On the other hand, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = (n_r - 1)v - \sum_{j \in N_r \cap L(i)} v/|L(j)| + v/|L(k)|$. Therefore, $\Phi^i(G) < \Phi^i(G')$ and agent i has an incentive to disconnect the link.

Subcase 2.2: When $R > 1$. In this case, $\phi_j^i(G)$ for $j \in N_r \setminus \{i\}$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(i) \setminus L(1)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) = (n_r - 1)v - \sum_{j \in L(i) \setminus L(1)} v/|L(j)|$. If $k \in L(1)$, then $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = \sum_{j \in N_r \setminus \{i\}} \phi_j^i(G)$ and we have $\Phi^i(G) = \Phi^i(G')$. If $k \notin L(1)$, then $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = \sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) + v/|L(k)|$ and we have $\Phi^i(G) < \Phi^i(G')$. Therefore, by Definition 4(i) of stability, agent i has an incentive to disconnect the link.

This concludes the proof for a nontrivial petal N_r such that $\{1\} \cup N_r$ is not a weak block. Now we turn to a nontrivial petal N_r such that $\{1\} \cup N_r$ is a weak block. Consider an agent $i \in \{1\} \cup N_r$ with at least 3 links in G_r, and let G' be the network when i disconnects one of her links, say link ik.

Case 1: $i = 1$.

We have $\phi_1^1(G) = \phi_1^1(G')$ by Lemma 4 and, in addition, $\phi_j^1(G) = \phi_j^1(G')$ for $j \notin N_r$ obviously. Therefore, $\sum_{j \notin N_r} \phi_j^1(G) = \sum_{j \notin N_r} \phi_j^1(G')$. Now, for $j \in N_r$, we consider two subcases.

Subcase 1.1: When $R = 1$. In this case, $\phi_j^1(G)$ for $j \in N_r$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(1)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r} \phi_j^1(G) = n_r v - \sum_{j \in N_r \cap L(1)} v/|L(j)|$. On the other hand, $\sum_{j \in N_r} \phi_j^1(G') = n_r v - \sum_{j \in N_r \cap L(1)} v/|L(j)| + v/|L(k)|$. Therefore, $\Phi^1(G) < \Phi^1(G')$ and agent 1 has an incentive to disconnect the link.

Subcase 1.2: When $R > 1$. In this case, $\phi_j^1(G)$ for $j \in N_r$ is equal to 0 if $j \in L(1)$ and is equal to Rv otherwise. Therefore, $\sum_{j \in N_r} \phi_j^1(G) = (n_r - |N_r \cap L(1)|)Rv$. On the other hand, $\sum_{j \in N_r} \phi_j^1(G') = (n_r - |N_r \cap L(1)|)Rv + Rv$. Therefore, $\Phi^1(G) < \Phi^1(G')$ and agent 1 has an incentive to disconnect the link.

Case 2: $i \neq 1$.

We will show that agent i has an incentive to disconnect the link ik, where we can safely assume that $k \neq 1$. First observe that $g_{1i} = 1$ if and only $g'_{1i} = 1$. Therefore, we have $\phi_i^i(G) = \phi_i^i(G')$ by Lemma 4. We also have $\phi_j^i(G) = \phi_j^i(G')$ for $j \notin N_r$. Therefore, $\sum_{j \notin N_r \setminus \{i\}} \phi_j^i(G) = \sum_{j \notin N_r \setminus \{i\}} \phi_j^i(G')$. Now, for $j \in N_r \setminus \{i\}$, we consider two subcases.

Subcase 2.1: When $R = 1$. In this case, $\phi_j^i(G)$ for $j \in N_r \setminus \{i\}$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(i)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) = (n_r - 1)v - \sum_{j \in N_r \cap L(i)} v/|L(j)|$. On the other hand, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = (n_r - 1)v - \sum_{j \in N_r \cap L(i)} v/|L(j)| + v/|L(k)|$. Therefore, $\Phi^i(G) < \Phi^i(G')$ and agent i has an incentive to disconnect the link.

Subcase 2.2: When $R > 1$. In this case, $\phi_j^i(G)$ for $j \in N_r \setminus \{i\}$ is equal to $(1 - 1/|L(j)|)v$ if $j \in L(i) \setminus L(1)$ and is equal to v otherwise. Therefore, $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) = (n_r - 1)v - \sum_{j \in L(i) \setminus L(1)} v/|L(j)|$. If $k \in L(1)$, then $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = \sum_{j \in N_r \setminus \{i\}} \phi_j^i(G)$ and we have $\Phi^i(G) = \Phi^i(G')$. If $k \notin L(1)$, then $\sum_{j \in N_r \setminus \{i\}} \phi_j^i(G') = \sum_{j \in N_r \setminus \{i\}} \phi_j^i(G) + v/|L(k)|$ and we have $\Phi^i(G) < \Phi^i(G')$. Therefore, by Definition 4(i) of stability, agent i has an incentive to disconnect the link. This concludes the proof of Proposition 3. $\diamondsuit$

Proof of Proposition 4. Suppose that G contains only one petal. By Proposition 3, G must be a circle network.[26] It is easy to see that $\Phi^i(G) = v$ for all $i \in N$. If agent i disconnects a link with another agent, then the network becomes a line network G'. It is also easy to see that $\Phi^i(G') = v$ for all $i \in N$. Therefore, i has an incentive to disconnect a link by Definition 4(i) of stability. $\diamondsuit$

[26] Recall that we assume $n \geq 4$.

Proof of Proposition 5. By Proposition 4, G contains at least two petals, that is, $R \geq 2$. Now consider a nontrivial petal N_r and let $G' = G - 1i$ be the network where agent 1 disconnect a link in N_r, say $1i$. We observe that, since every nontrivial petal constitutes a circle network by Proposition 3, $G'_r = G_r - 1i$ is a line network. Then, we have $\Phi^1(G) = \frac{1}{n}[(R+1)v + (n_r - 2)Rv + \sum_{j \notin \{1\} \cup N_r} \phi_j^1(G)]$ and $\Phi^1(G') = \frac{1}{n}[Rv + n_r v + \sum_{j \notin \{1\} \cup N_r} \phi_j^1(G')]$. Since Lemma 1(v) implies that $\phi_j^1(G') \geq \phi_j^1(G)$ for all $j \notin \{1\} \cup N_r$, we have $\Phi^1(G') - \Phi^1(G) \geq \frac{1}{n}[(n_r - 1)v - (n_r - 2)Rv]$. When (i) $n_r = 2$ or (ii) $R = 2$ and $n_r = 3$, agent 1 has an incentive to disconnect since $\Phi^1(G') - \Phi^1(G) \geq 0$. $\diamondsuit$

Proof of Proposition 6. We first show that if G is stable then it contains at least one spoke. Suppose to the contrary that G does not contain a spoke. Then G contains two or more nontrivial petals each of which constitutes a circle network by Propositions 3 and 4. Now consider a nontrivial petal N_r and let $G' = G - 1i$ is the network where agent 1 disconnect a link in G_r, say $1i$. Observe that, for $j \notin \{1\} \cup N_r$, Lemma 1(v) implies that $\phi_j^1(G) = \phi_j^1(G') = 0$ if $j \in L(1)$; and $\phi_j^1(G) = Rv$ and $\phi_j^1(G') = (R-1)v + n_r v$ if $j \notin L(1)$. Let x be the number of agents who do not belong to $\{1\} \cup N_r$ and who are not linked to agent 1. Then, the preceding observation together with the derivation in the proof of Proposition 5 implies that $\Phi^1(G) = \frac{1}{n}[(R+1)v + (n_r - 2)Rv + xRv]$ and $\Phi^1(G') = \frac{1}{n}[(R+n_r)v + x(R+n_r-1)v]$. Hence, $\Phi^1(G') - \Phi^1(G) = \frac{1}{n}[(x+1)(n_r-1)v - (n_r-2)Rv]$. Since each nontrivial petal has at least 3 agents by Proposition 5 and there exist only nontrivial petals by supposition, we have $x \geq R-1$, implying that $\Phi^1(G') - \Phi^1(G) \geq Rv/n$. Therefore, agent 1 has an incentive to disconnect the link $1i$.

We next show that if G is stable then it cannot contain more than one spoke. Suppose to the contrary that G contains two spokes $N_r = \{i\}$ and $N_{r'} = \{k\}$. It is easy to see that $\Phi^i(G) = (R+1)v/n$ since $\phi_i^i(G) = (R+1)v$ by Lemma 1(i)-(ii) and $\phi_j^i(G) = 0$ for all $j \neq i$ by Lemma 1(iii)-(iv). On the other hand, in the new network $G' = G + ik$ where agents i and k establish a link, we have $\phi_i^i(G') = Rv$, $\phi_k^i(G') = v$ and $\phi_j^i(G') = v/2$ for all

$j \neq i, k$.[27] Therefore, $\Phi^i(G') = \frac{1}{n}[(R+1)v + (n-2)v/2] > \Phi^i(G)$. Since agent k faces a symmetric situation, we conclude that agents i and k have an incentive to establish a link. $\diamondsuit$

Proof of Proposition 7. Proposition 6 says that G contains exactly one spoke, say $\{i\}$. Proposition 4 then implies that G must contain at least one nontrivial petal. Now suppose that G contains more than one nontrivial petal, say N_r and $N_{r'}$. Proposition 5 implies that $n_r \geq 3$ and $n_{r'} \geq 3$. Consider the new network $G' = G + ik$ where agent k is an agent in N_r who has a link with agent 1. We note that N_r constitutes a circle network by Proposition 3 and the merged petal $\{i\} \cup N_r$ is a nontrivial petal. It is easy to see that $\phi_i^i(G') = Rv$ and $\phi_j^i(G') = v$ for all $j \in N_r$. Therefore, $\Phi^i(G') \geq (R+3)v/n > (R+1)v/n = \Phi^i(G)$.

We now turn to agent k's incentive. In the original network G, we have $\phi_k^k(G) = (R+1)v$ and $\phi_j^k(G) = v/2$ for all $j \notin N_r$ by Lemma 1. In addition, $\sum_{j \in N_r \setminus \{k\}} \phi_j^k(G)$ is equal to $(n_r - 1)v - v/2$. Therefore, $\Phi^k(G) = \frac{1}{n}[(R+1)v + (n - n_r)v/2 + (n_r - 1)v - v/2]$. In the new network $G' = G + ik$, we have $\phi_k^k(G') = Rv$, $\phi_i^k(G') = v$ and $\phi_j^k(G') = 2v/3$ for all $j \notin \{i\} \cup N_r$. It is also easy to see that $\sum_{j \in N_r \setminus \{k\}} \phi_j^k(G') = \sum_{j \in N_r \setminus \{k\}} \phi_j^k(G) = (n_r - 1)v - v/2$. Therefore, $\Phi^k(G') = \frac{1}{n}[(R+1)v + (n - n_r - 1)2v/3 + (n_r - 1)v - v/2]$. We thus get $\Phi^k(G') - \Phi^k(G) = \frac{1}{n}[(n - n_r - 4)v/6]$ and, since $n \geq 2 + n_r + n_{r'}$ and $n_{r'} \geq 3$, we have $n - n_r - 4 \geq n_{r'} - 2 > 0$. We conclude that agents i and k have an incentive to establish a link. $\diamondsuit$

Proof of Proposition 8. Consider a simple network G. Proposition 5 implies that the nontrivial petal of G has at least four agents. Suppose now to the effect of contradiction that the nontrivial petal has more than four agents and, for $n > 6$, we have $g_{1,2} = g_{2,3} = \cdots = g_{n-2,n-1} = g_{n-1,1} = 1$ and $g_{1,n} = 1$ without loss of generality. That is, $\{2, \ldots, n-1\}$ is the nontrivial petal and $\{n\}$ is the spoke of G. We will show that agent 4 has an incentive to disconnect a link. In the original network G, it is easy to see that $\Phi^4(G) = v$. On the

[27] Recall that we assume $n \geq 4$.

other hand, if agent 4 disconnects a link with agent 5, then she still enjoys $\Phi^4(G') = v$ in the new network $G' = G - 45$. Therefore, by Definition 4(i) of stability, agent 4 has an incentive to disconnect the link. $\diamondsuit$

Proof of Proposition 10. Consider first a closed line segment of length > 4 and denote it by G. We have $\Phi^i(G) = \frac{1}{m}[v^1 + v^m + (m-2)v]$ for all $i = 1, \ldots, m$. On the other hand, in the new network $G' = G + 1m$ where agents 1 and m establish a link, we have $\Phi^1(G') = \frac{1}{m}[v^1 + v^m + (m-3)v^1] > \Phi^1(G)$ since $v^1 \geq 2v$ and $m > 4$. Likewise, we have $\Phi^m(G') = \frac{1}{m}[v^1 + v^m + (m-3)v^m] > \Phi^m(G)$. Therefore, a closed line segment with $m > 4$ is not stable.

Consider next an open line segment of length > 4 and denote it again by G. We have $\Phi^i(G) = \frac{1}{m}[v^1 + (m-1)v]$ for all $i = 1, \ldots, m$. On the other hand, in the new network $G' = G + 2m$ where agents 2 and m establish a link, we have $\Phi^2(G') = \frac{1}{m}[v^1 + 2v + (m-4)(v^1 + v)] > \Phi^2(G)$ since $m > 4$. We also have $\Phi^m(G') = \frac{1}{m}[v^1 + (m-1)v + v/2] > \Phi^m(G)$. Therefore, an open line segment with $m > 4$ is not stable. $\diamondsuit$

Proof of Proposition 11. It is straightforward to check that line networks of length $n \leq 5$ are stable. For $n > 5$, consider a line network G where $g_{1,2} = g_{2,3} = \cdots = g_{n-1,n} = 1$ without loss of generality. We have $\Phi^i(G) = v$ for all $i = 1, \ldots, n$. On the other hand, in the new network $G' = G + 2n$ where agents 2 and n establish a link, we have $\Phi^2(G') = \frac{1}{n}[3v + (n-4)2v] = \frac{1}{n}[nv + (n-5)v] > \Phi^2(G)$ since $n > 5$. We also have $\Phi^n(G') = \frac{1}{n}[nv + v/2] > \Phi^n(G)$. Therefore, line networks with $n > 5$ are not stable. $\diamondsuit$

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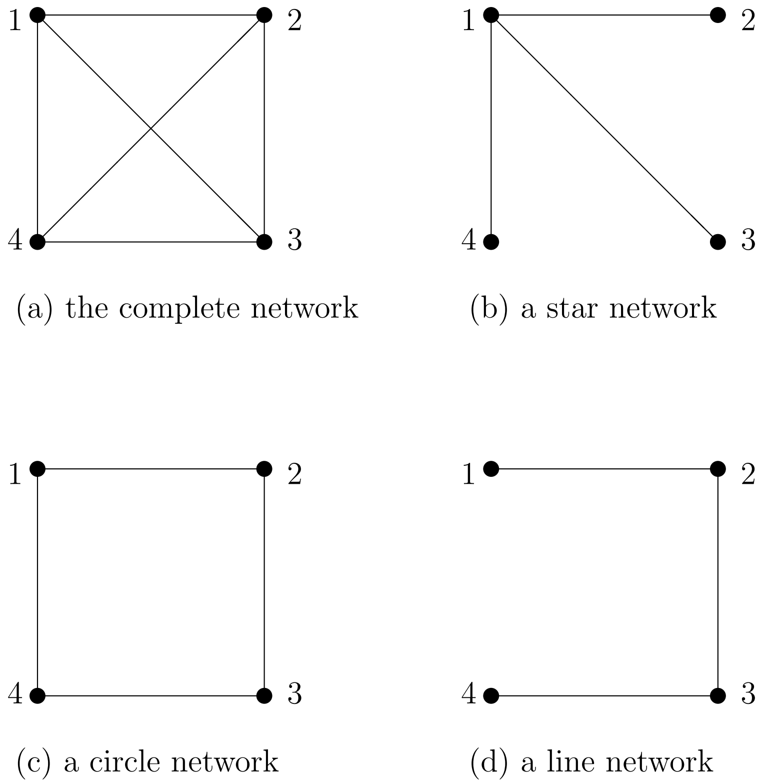


(a) the complete network (b) a star network

(c) a circle network (d) a line network

Figure 1: Some Specific Networks

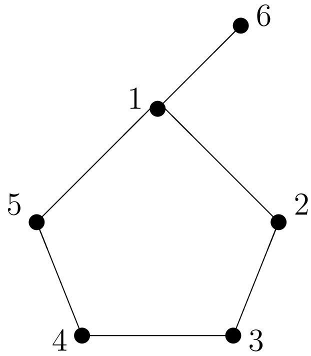
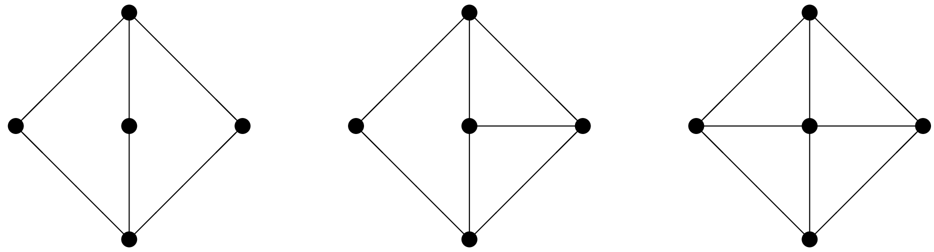
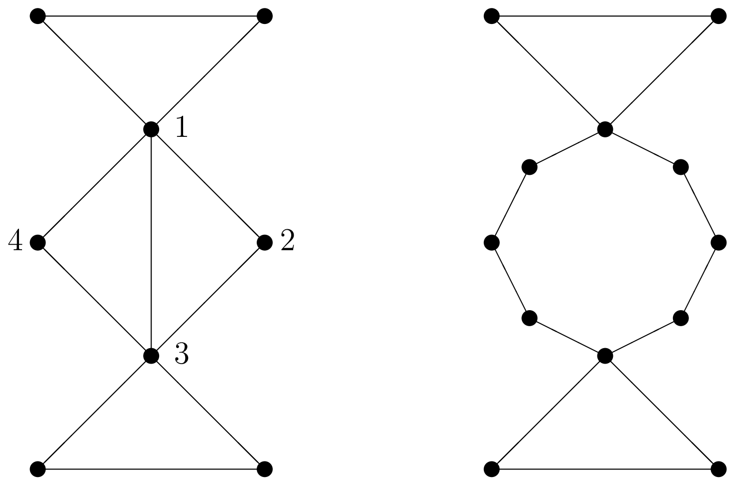


Figure 2: A Stable Simple Network



(a) a weak block (b) a semi-strong block (c) a strong block

Figure 3: Classification of Blocks



(a) (b)

Figure 4: Two Examples of General Networks

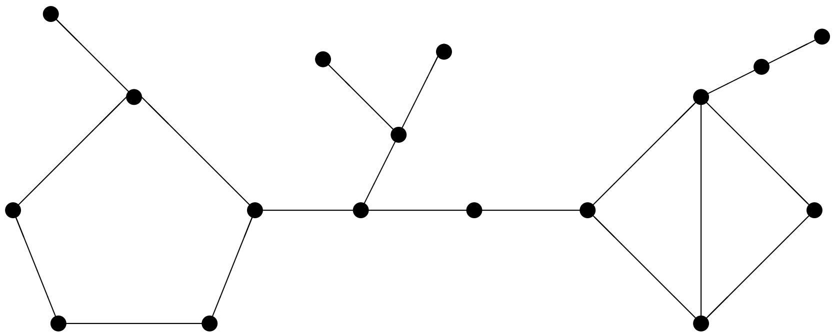


Figure 5: A Possible Stable Network