Robust Double Auction Mechanisms[*]

Kiho Yoon[†]

Abstract   We study the robust double auction mechanisms, that is, the double auction mechanisms that satisfy dominant strategy incentive compatibility, ex-post individual rationality and ex-post budget balance. We first establish that the price in any robust mechanism does not depend on the valuations of the trading players. We next establish that, with a non-bossiness assumption, the price in any robust mechanism does not depend on players' valuations at all, whether trading or non-trading. Our main result is the characterization result that, with a non-bossy assumption along with other assumptions on the properties of the mechanism, the generalized posted mechanism in which a constant price is posted for each possible set of traders is the only robust double auction mechanism.

Keywords   Double auction, posted price, robust mechanism design, dominant strategy, budget balance

JEL Classification   C72, D47, D82

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1. INTRODUCTION

Double auctions are widely used in many two-sided market situations such as stock exchanges as well as commodity markets. In a typical double auction, (i) there are many sellers and many buyers who have private information about their true valuations for the good, (ii) they simultaneously submit their respective offers and bids to the mechanism (or the center, the mediator, the clearing house, the social planner, etc.), and (iii) the mechanism determines the volume and the terms of trade according to a pre-determined rule. Double auctions approximate many other trading settings in the real world. They also have great theoretical value in the study of price formation under private information.

In this paper, we study the double auction mechanisms that satisfy dominant strategy incentive compatibility, ex-post individual rationality and ex-post budget balance. We call this family of double auction mechanisms as *robust* double auction mechanisms. A special instance of the robust double auction mechanisms is the robust bilateral trading mechanism for the environments in which there is one seller who initially owns one indivisible good and there is one buyer who is interested in obtaining the good. Hagerty and Rogerson (1987) and Čopič and Ponsatí (2016)have shown that the only robust bilateral trading mechanism is the posted price mechanism, i.e., the mechanism in which a price is posted in advance and the seller and the buyer either trade at this price or do not trade at all. The purpose of this paper is to obtain a similar characterization for the general robust double auction mechanisms.

In the next section, we first establish that the price in any robust mechanism does not depend on the valuations of the *trading* players. This result in particular implies that a robust bilateral trading mechanism is a posted price mechanism. In contrast, we show that a robust double auction mechanism may not be a posted price mechanism when there are more than two players.[1] We next establish that, with a non-bossiness assumption, the price in any robust mechanism does not depend on players' valuations at all, whether trading or non-trading. Our main result is the characterization result for the general double auction environments that, with a non-bossy assumption along with other assumptions on the properties of the mechanism, the generalized posted mechanism in which a constant price is posted for each possible set of traders is the only robust double auction mechanism. Section 3 concludes with some discussion on limitation and future research.

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[1]Hence, Hagerty and Rogerson's conjecture (in page 97 of their paper) that the posted price mechanism may be the only robust mechanism in 'n-person trading problems' is incorrect.

There is a large literature on double auction mechanisms: Representative
works include Wilson (1985), Rustichini *et al.* (1994), Satterthwaite and Williams
(2002), Cripps and Swinkels (2006) and Satterthwaite *et al.* (2014). Most of the
papers in this literature examine double auction mechanisms under the Bayesian
incentive compatibility postulate. In contrast, the current paper focuses on robust
double auction mechanisms. Robustness is desirable since the equilibrium be-
havior does not depend on the fine details of the environments, such as players'
beliefs about each other. We note that the only other double auction mechanisms
that satisfy dominant strategy incentive compatibility is McAfee (1992) mech-
anism and Yoon (2001) modified Vickrey double auction mechanism, but these
mechanisms do not satisfy ex-post budget balance.

The papers similar in spirit but examining settings quite different from ours
are Barber and Jackson (1995) and Miyagawa (2001). The former studies classi-
cal exchange economies with many goods and shows that the only robust mecha-
nism, under some additional conditions, is the fixed-proportion anonymous trad-
ing mechanism. The latter studies housing markets (i.e., trading situations in
which each player initially owns one object to trade) with money and shows that
the only robust mechanism, under some additional conditions, is the fixed-price
core mechanism.

The research on robust mechanism design is not limited to the trading set-
tings. Recently, Drexl and Kleiner (2015), Shao and Zhou (2016) and Yoon
(2018) study the robust allocation problem of assigning one indivisible private
good to one of the players, whereas Kuzmics and Steg (2017) study the robust
public good provision problem. On a more general level, there is a voluminous
literature concerning the common knowledge assumption and robust mechanism
design after the pioneering work of Bergemann and Morris (2005).[2]

2. THE CHARACTERIZATION

There is a set $\mathscr{S} = \{1, \ldots, m\}$ of sellers and a set $\mathscr{B} = \{1, \ldots, n\}$ of buyers
for a good. Each seller owns one indivisible unit of the good to sell, and each
buyer wants to buy at most one unit of the good. Hence, there are m units of
the good available. Seller i's privately known valuation for the good is denoted
by s_i, and buyer j's privately known valuation for the good is denoted by b_j.
We assume that $s_i \in [0,1]$ for all $i \in \mathscr{S}$ as well as $b_j \in [0,1]$ for all $j \in \mathscr{B}$. Let
$s = (s_1, \ldots, s_m)$ and $b = (b_1, \ldots, b_n)$. In addition, let $v = (s,b)$ and $V = [0,1]^{m+n}$.
Hence, V is the set of possible valuation profiles of the sellers and the buyers.

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[2] See Bergemann and Morris (2013) for an excellent introduction.

We use the usual notation such as $v = (s_i, s_{-i}, b) = (s, b_j, b_{-j})$. We also use $v = (v_i, v_{-i})$ for $i \in \mathscr{S}$ and $v = (v_j, v_{-j})$ for $j \in \mathscr{B}$. That is, $(v_i, v_{-i}) = (s_i, s_{-i}, b)$ for $i \in \mathscr{S}$ where $v_i = s_i$ and $v_{-i} = (s_{-i}, b)$, and $(v_j, v_{-j}) = (s, b_j, b_{-j})$ for $j \in \mathscr{B}$ where $v_j = b_j$ and $v_{-j} = (s, b_{-j})$.

Let $p_i : V \to [0, 1]$ be the allocation rule for seller $i \in \mathscr{S}$, and let $q_j : V \to [0, 1]$ be the allocation rule for buyer $j \in \mathscr{B}$. Both $p_i(v)$ and $q_j(v)$ denote the probability of trade. In addition, let $x_i : V \to \mathbb{R}$ be the transfer rule for seller $i \in \mathscr{S}$ such that $x_i(v)$ is the receipt of the money seller i gets, and let $y_j : V \to \mathbb{R}$ be the transfer rule for buyer $j \in \mathscr{B}$ such that $y_j(v)$ is the payment of the money buyer j makes. Note that $x_i(v)$ may be positive or negative: when it is negative, seller i pays that amount. Likewise, $y_j(v)$ may be positive or negative: when it is negative, buyer j receives that amount. The payoff of seller $i \in \mathscr{S}$ and buyer $j \in \mathscr{B}$ is denoted respectively by

$$u_i(v) = x_i(v) - p_i(v)s_i \quad \text{and} \quad u_j(v) = q_j(v)b_j - y_j(v).$$

Let $p = (p_1, \ldots, p_m), q = (q_1, \ldots, q_n), x = (x_1, \ldots, x_m)$, and $y = (y_1, \ldots, y_n)$. The mechanism (p, q, x, y) is said to be *dominant strategy incentive compatible* if

$$\begin{aligned} &u_i(s, b) \geq x_i(s'_i, s_{-i}, b) - p_i(s'_i, s_{-i}, b)s_i \ \ \forall i \in \mathscr{S}, \forall s_i, \forall s'_i, \forall s_{-i}, \forall b; \\ &u_j(s, b) \geq q_j(s, b'_j, b_{-j})b_j - y_j(s, b'_j, b_{-j}) \ \ \forall j \in \mathscr{B}, \forall b_j, \forall b'_j, \forall b_{-j}, \forall s \end{aligned} \quad (IC)$$

and *ex-post individually rational* if

$$u_i(v) \geq 0 \ \ \forall i \in \mathscr{S} \ \text{ and } \ u_j(v) \geq 0 \ \ \forall j \in \mathscr{B}, \ \ \forall v \in V. \quad (IR)$$

The mechanism (p, q, x, y) is said to be *ex-post budget balancing* if

$$\sum_{i \in \mathscr{S}} x_i(v) = \sum_{j \in \mathscr{B}} y_j(v) \ \ \forall v \in V \quad (BB)$$

and *non-wasteful* if

$$\sum_{i \in \mathscr{S}} p_i(v) = \sum_{j \in \mathscr{B}} q_j(v) \ \ \forall v \in V. \quad (NW)$$

Note well that we may have $\sum_{i \in \mathscr{S}} p_i(v) > \sum_{j \in \mathscr{B}} q_j(v)$ if the mechanism withholds/destroys some units. We exclude this possibility. We will call the mechanisms that satisfy (IC), (IR), (BB) and (NW) as *robust double auction mechanisms*. We first have the following standard lemma.

Lemma 1. *The mechanism (p, q, x, y) is dominant strategy incentive compatible (IC) if and only if*

(i) $\forall i \in \mathscr{S}, \forall v_{-i}$: $p_i(v)$ *is weakly decreasing in* v_i.

(ii) $\forall j \in \mathscr{B}, \forall v_{-j}$: $q_j(v)$ *is weakly increasing in* v_j.

(iii) $\forall i \in \mathscr{S}, \forall v$: $u_i(v) = \int_{v_i}^1 p_i(w, v_{-i})dw + u_i(1, v_{-i})$.

(iv) $\forall j \in \mathscr{B}, \forall v$: $u_j(v) = \int_0^{v_j} q_j(w, v_{-j})dw + u_j(0, v_{-j})$.

Proof: Omitted since it is standard. See Myerson (1981), Myerson and Satterthwaite (1983), etc.   *Q.E.D.*

We will restrict our attention to the deterministic allocation rules that take only the value zero or one.[3] Thus, $p_i : V \to \{0,1\}$ for all $i \in \mathscr{S}$ and $q_j : V \to \{0,1\}$ for all $j \in \mathscr{B}$. Let $S(v) = \{i \in \mathscr{S}|p_i(v) = 1\}$ denote the set of trading sellers at v, and let $B(v) = \{j \in \mathscr{B}|q_j(v) = 1\}$ denote the set of trading buyers at v.

A possible property that the mechanism may satisfy is that all sellers face one identical price and all buyers face another identical price.

Assumption 1. (Common price) $\forall v \in V, \forall \{i, i'\} \subseteq S(v), \forall \{j, j'\} \subseteq B(v)$:
$$x_i(v) = x_{i'}(v) \text{ and } y_j(v) = y_{j'}(v).$$

Hence, there are two prices, the seller price (ask price) of $\pi^s(v)$ and the buyer price (bid price) of $\pi^b(v)$. Of course, these two prices may be the same. Note that $\pi^s(v)$ and $\pi^b(v)$ may vary with v. This assumption is reasonable since all units of the good are identical and the players supply or demand at most one unit. Observe in particular that, though the famous VCG (Vickrey-Clarke-Groves) mechanisms may in general induce different transfers of money across players, there is only one seller price when each seller supplies one unit and only one buyer price when each buyer demands at most one unit. In fact, this assumption seems to be a prerequisite for any centralized trading institution. Example 1 below shows that we may pair one seller and one buyer and let the trade occur only between them. Then, each seller-buyer pair is a separated market and different prices across the pairs may emerge. Although we do not explicitly consider decentralized trading or resale possibilities in the current mechanism design approach, if the buyers face different prices, say, then it is conceivable that a buyer of the more expensive unit could find a new trade opportunity with a seller to the advantage of both.

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[3]It certainly entails loss of generality to restrict the attention to deterministic allocation rules. See the last section for further discussion. We want to add that any tie-breaking rule in the subsequent deterministic allocation rule is sufficient for our characterization.

Another possible property that the mechanism may satisfy is that the payoff of a player with the worst possible valuation is equal to zero: Observe that $u_i(v)$ is lowest when $v_i = 1$ and $u_j(v)$ is lowest when $v_j = 0$ by Lemma 1(iii)-(iv).

Assumption 2. (Zero payoff for the worst type)
$$\forall i \in \mathscr{S}, \forall v_{-i} : u_i(1, v_{-i}) = 0 \text{ and } \forall j \in \mathscr{B}, \forall v_{-j} : u_j(0, v_{-j}) = 0.$$

Note that the condition $u_j(0, v_{-j}) = 0$ for the buyer $j \in \mathscr{B}$ holds if and only if $y_j(0, v_{-j}) = 0$ holds. The latter condition is often assumed in the literature. Observe that (IR) implies $y_j(0, v_{-j}) \leq 0$ whereas the no subsidy condition implies $y_j(0, v_{-j}) \geq 0$, leading us to $y_j(0, v_{-j}) = 0$ and hence $u_j(0, v_{-j}) = 0.$[4] The seller case is essentially symmetric. Observe that (IR) implies $x_i(1, v_{-i}) \geq p_i(1, v_{-i})$ whereas the no subsidy condition implies $x_i(1, v_{-i}) \leq p_i(1, v_{-i})$, leading us to $u_i(1, v_{-i}) = 0$.

We now characterize some necessary conditions that a robust double auction mechanism must satisfy. We use the notation $|A|$ to denote the cardinality of an arbitrary set A, and the notation v_{-T} to denote the valuation profile of the players not in the set $T \subseteq \mathscr{S} \cup \mathscr{B}$.

Proposition 1. *Let the mechanism (p,q,x,y) be a robust double auction mechanism. Under Assumption 1 of common price and Assumption 2 of zero payoff for the worst type, the mechanism (p,q,x,y) must be the following form: for all $v \in V$ and $T(v) = S(v) \cup B(v)$,*

(i) $|S(v)| = |B(v)|$;

(ii) $x_i(v) = y_j(v) = \pi(v_{-T(v)})$ for all $i \in S(v)$ and for all $j \in B(v)$;

(iii) $x_i(v) = y_j(v) = 0$ for all $i \notin S(v)$ and for all $j \notin B(v)$;

(iv) $v_i \leq \pi(v_{-T(v)})$ for all $i \in S(v)$ and $v_j \geq \pi(v_{-T(v)})$ for all $j \in B(v)$.

Remark: Property (i) says that the number of trading sellers must be equal to the number of trading buyers. Property (ii) says that all trading sellers and trading buyers face the same price $\pi(v_{-T(v)})$ that is independent of their valuations. Note in particular that there is only one price, i.e., the seller price is identical to the buyer price. Property (iii) says that the sellers and the buyers who do not trade neither receive nor pay any money. Property (iv) says that every trading seller's valuation is less than or equal to the price and every trading buyer's valuation is greater than or equal to the price.

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[4]The no subsidy condition is (i) $\forall j \in \mathscr{B}, \forall v : y_j(v) \geq 0$, and (ii) $\forall i \in \mathscr{S}, \forall v : x_i(v) \leq p_i(v)$.

Proof: It is obvious that (NW) implies property (i). Next, since $p_i(v_i, v_{-i}) \in \{0, 1\}$, Lemma 1(i) implies: for a given v_{-i}, there exists $z_i(v_{-i}) \in [0, 1]$ such that

$$p_i(v_i, v_{-i}) = \begin{cases} 1 & \text{if } v_i < z_i(v_{-i}); \\ 0 & \text{if } v_i > z_i(v_{-i}). \end{cases}$$

Likewise, Lemma 1(ii) implies: for a given v_{-j}, there exists $z_j(v_{-j}) \in [0, 1]$ such that

$$q_j(v_j, v_{-j}) = \begin{cases} 0 & \text{if } v_j < z_j(v_{-j}); \\ 1 & \text{if } v_j > z_j(v_{-j}). \end{cases}$$

That is,

$$z_i(v_{-i}) = \sup\{v_i \in [0, 1] \,|\, p_i(v_i, v_{-i}) = 1\} \text{ for } i \in \mathscr{S};$$

$$z_j(v_{-j}) = \inf\{v_j \in [0, 1] \,|\, q_j(v_j, v_{-j}) = 1\} \text{ for } j \in \mathscr{B}.$$

Then, by Lemma 1(iii) and Assumption 2,

$$u_i(v) = \begin{cases} z_i(v_{-i}) - v_i & \text{when } p_i(v_i, v_{-i}) = 1; \\ 0 & \text{when } p_i(v_i, v_{-i}) = 0. \end{cases}$$

Likewise, by Lemma 1(iv) and Assumption 2,

$$u_j(v) = \begin{cases} v_j - z_j(v_{-j}) & \text{when } q_j(v_j, v_{-j}) = 1; \\ 0 & \text{when } q_j(v_j, v_{-j}) = 0. \end{cases}$$

Hence, we have:

(i) For a seller $i \in \mathscr{S}$:

$$x_i(v) = \begin{cases} z_i(v_{-i}) & \text{when } p_i(v) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

(ii) For a buyer $j \in \mathscr{B}$:

$$y_j(v) = \begin{cases} z_j(v_{-j}) & \text{when } q_j(v) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

We then have

$$\sum_{i \in S(v)} x_i(v) = \sum_{j \in B(v)} y_j(v) \quad \forall v \in V$$

by (BB) and the fact that $x_i(v) = 0$ for all $i \notin S(v)$ and $y_j(v) = 0$ for all $j \notin B(v)$. Since $\sum_{i \in S(v)} x_i(v) = |S(v)| \pi^s(v)$ and $\sum_{j \in B(v)} y_j(v) = |B(v)| \pi^b(v)$ by Assumption

1 and $|S(v)| = |B(v)|$ by (i), we have $\pi^s(v) = \pi^b(v)$. Let us denote this price by $\pi(v)$.

If $T(v) \neq \emptyset$, then $\pi(v) = z_k(v_{-k})$ for all $k \in T(v)$. Hence, $\pi(v)$ does not depend on v_k for any $k \in T(v)$. So, we can write this price as $\pi(v_{-T(v)})$ for all $v \in V$ and $T(v) \subseteq \mathscr{S} \cup \mathscr{B}$. This proves that properties (ii) and (iii) hold. It is now clear that (IR) implies property (iv).   *Q.E.D.*

Both Assumption 1 and Assumption 2 are essential for Proposition 1 as the following examples demonstrate. Note that Assumption 2 is satisfied in Example 1 whereas Assumption 1 is (trivially) satisfied in Example 2.

Example 1. Suppose there are two sellers and two buyers, and consider a mechanism in which (i) two distinct constant prices $\pi_1^s = \pi_1^b$ and $\pi_2^s = \pi_2^b$ are posted in advance, and (ii) a trade occurs between seller 1 and buyer 1 when and only when $s_1 < \pi_1^s = \pi_1^b < b_1$ and a trade occurs between seller 2 and buyer 2 when and only when $s_2 < \pi_2^s = \pi_2^b < b_2$. Hence, seller 1 and buyer 1 face the posted price $\pi_1^s = \pi_1^b$ and trade only between them, and likewise for seller 2 and buyer 2. This mechanism is a robust double auction mechanism that violates Assumption 1. In this example, there are two posted price mechanisms separately applied to two pairs of traders, the pair of seller 1 and buyer 1 and the pair of seller 2 and buyer 2.

Example 2. Suppose there are one seller and two buyers, and consider a mechanism in which (i) two prices, the seller price π^s and the buyer price π^b, are posted in advance with $\pi^s < \pi^b$, (ii) a trade occurs between seller 1 and buyer 1 when and only when $s_1 < \pi^s < \pi^b < b_1$, (iii) when a trade occurs, seller 1 receives π^s, buyer 1 pays π^b, and buyer 2 receives $\pi^b - \pi^s$, that is, $x_1(v) = \pi^s$, $y_1(v) = \pi^b$, and $y_2(v) = \pi^s - \pi^b$, and (iv) when there is no trade, every player's monetary transfer is equal to zero, that is, $x_1(v) = y_1(v) = y_2(v) = 0$. This mechanism is a robust double auction mechanism without one price for all traders. The reason Proposition 1 does not hold in this example is that Assumption 2 is not satisfied.[5]

Observe that, for bilateral trading with one seller and one buyer, i.e., $\mathscr{S} = \mathscr{B} = \{1\}$, we always have $T(v) = \mathscr{S} \cup \mathscr{B}$ unless $T(v) = \emptyset$. Hence, $\pi(v)$ does not depend on either s_1 or b_1 whenever $T(v) \neq \emptyset$. This mechanism can be implemented by a posted price mechanism: A constant price π is posted in advance and the trade occurs when and only when $s_1 < \pi < b_1$.[6] Since it is straightfor-

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[5]We note that we can construct a similar mechanism for the case when there are more than one seller and more than one buyer.

[6]We assume that players decide not to trade when they are indifferent between trade and no

ward to show that Assumptions 1 and 2 are satisfied for bilateral trading environments, Proposition 1 implies one of the results in Hagerty and Rogerson (1987) that a robust double auction mechanism for bilateral trading environments is a posted price mechanism.

Corollary 1. *A double auction mechanism for bilateral trading environments is robust if and only if it is a posted price mechanism.*

Proof: Observe that Assumption 1 is trivially satisfied for bilateral trading environments. Observe next that $x_1(s_1,b_1) = y_1(s_1,b_1)$ by (BB) and $p_1(s_1,b_1) = q_1(s_1,b_1)$ by (NW). Hence, the total surplus, i.e., the sum of players' payoffs, is $p_1(s_1,b_1)(b_1 - s_1)$. This becomes (i) $p_1(1,b_1)(b_1 - 1) \leq 0$ when $s_1 = 1$, and (ii) $-p_1(s_1,0)s_1 \leq 0$ when $b_1 = 0$. Since the total surplus is nonpositive when either $s_1 = 1$ or $b_1 = 0$ whereas each player's payoff is nonnegative by (IR), the only possibility is that each player gets a zero payoff. Thus, Assumption 2 is always satisfied for bilateral trading environments. Proposition 1 then implies that a mechanism in these environments is robust only if it is a posted price mechanism. The converse also holds since a posted price mechanism is clearly robust. *Q.E.D.*

In contrast, a robust double auction mechanism may not be a posted price mechanism when there are more than two players, as the following example demonstrates.

Example 3. Suppose there are one seller and two buyers, and consider a mechanism in which (i) players first submit their valuations (which may or may not be true valuations), (ii) the price is set to the lower of the buyers' reported valuations, and (iii) a trade occurs between the seller and the buyer of the higher valuation when and only when the seller's valuation is less than the price. When $s_1 = 0.3$, $b_1 = 0.7$, and $b_2 \in (0.3, 0.7)$, for instance, the trade occurs and the price $\pi = b_2$ depends on buyer 2's valuation.

Observe in particular that the mechanism in this example is dominant strategy incentive compatible (IC), mainly due to the fact that the traders cannot affect the terms of trade. On the other hand, the non-trading buyer affects the terms of trade without altering her own allocation or payment. We now introduce another property to obtain a sharper result. The mechanism (p,q,x,y) is said to be *non-bossy in payments* if

(i) For all $i \in \mathscr{S}$ and for all v_i, v_i', and v_{-i}: if $p_i(v_i, v_{-i}) = p_i(v_i', v_{-i})$ and $x_i(v_i, v_{-i}) = x_i(v_i', v_{-i})$, then $x(v_i, v_{-i}) = x(v_i', v_{-i})$ and $y(v_i, v_{-i}) = y(v_i', v_{-i})$.

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trade. We follow this convention throughout the paper.

(ii) For all $j \in \mathscr{B}$ and for all v_j, v_j', and v_{-j}: if $q_j(v_j, v_{-j}) = q_j(v_j', v_{-j})$ and $y_j(v_j, v_{-j}) = y_j(v_j', v_{-j})$, then $x(v_j, v_{-j}) = x(v_j', v_{-j})$ and $y(v_j, v_{-j}) = y(v_j', v_{-j})$.

In words, a mechanism is non-bossy in payments if no player can change others' payments without changing his/her allocation or payment. We note that this definition is different from the standard one first introduced by Satterthwaite and Sonnenschein (1981). The standard non-bossiness property is stated as follows in the current setup: the mechanism (p, q, x, y) is non-bossy if (i) for all $i \in \mathscr{S}$ and for all v_i, v_i', and v_{-i}: if $p_i(v_i, v_{-i}) = p_i(v_i', v_{-i})$ and $x_i(v_i, v_{-i}) = x_i(v_i', v_{-i})$, then $p(v_i, v_{-i}) = p(v_i', v_{-i}), q(v_i, v_{-i}) = q(v_i', v_{-i}), x(v_i, v_{-i}) = x(v_i', v_{-i})$ and $y(v_i, v_{-i}) = y(v_i', v_{-i})$; (ii) for all $j \in \mathscr{B}$ and for all v_j, v_j', and v_{-j}: if $q_j(v_j, v_{-j}) = q_j(v_j', v_{-j})$ and $y_j(v_j, v_{-j}) = y_j(v_j', v_{-j})$, then $p(v_j, v_{-j}) = p(v_j', v_{-j}), q(v_j, v_{-j}) = q(v_j', v_{-j})$, $x(v_j, v_{-j}) = x(v_j', v_{-j})$ and $y(v_j, v_{-j}) = y(v_j', v_{-j})$. That is, the conclusion part includes the statements about allocations as well as payments.[7] Hence, our definition is weaker than the standard definition.

Assumption 3. (Non-bossiness-in-payments) The mechanism (p, q, x, y) is non-bossy in payments.

This non-bossiness-in-payments assumption is an extra property imposed on the mechanism on top of the properties required by Proposition 1 under which the generalized posted price mechanism results. We note that the non-bossiness assumption in the standard form is also imposed (along with additional assumptions) in the works of Barber and Jackson (1995) and Miyagawa (2001) cited in the introduction. We have:

Proposition 2. *Let the mechanism (p, q, x, y) be a robust double auction mechanism. Under Assumption 1 of common price, Assumption 2 of zero payoff for the worst type and Assumption 3 of non-bossiness-in-payments, the mechanism (p, q, x, y) must be the following form: for all $v \in V$ and $T(v) = S(v) \cup B(v)$, there exists a constant price π such that*

(i) $|S(v)| = |B(v)|$;

(ii) $x_i(v) = y_j(v) = \pi$ for all $i \in S(v)$ and for all $j \in B(v)$;

(iii) $x_i(v) = y_j(v) = 0$ for all $i \notin S(v)$ and for all $j \notin B(v)$;

(iv) $v_i \leq \pi$ for all $i \in S(v)$ and $v_j \geq \pi$ for all $j \in B(v)$.

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[7] See Thomson (2016) for a thorough discussion of non-bossiness.

Proof: We know by Proposition 1(ii) that the price does not depend on the valuation profile of the traders, i.e., $\pi(v) = \pi(v_{-T(v)})$ for all $v \in V$. We now show that it does not depend on the valuation profile of the non-traders, either. Let there be a non-trading seller $i \in S^c(v_i, v_{-i}) \cap S^c(v_i', v_{-i})$. This means $p_i(v_i, v_{-i}) = 0 = p_i(v_i', v_{-i})$, and $x_i(v_i, v_{-i}) = 0 = x_i(v_i', v_{-i})$ by Proposition 1(iii). We have $\pi(v_i, v_{-T(v) \setminus \{i\}}) = \pi(v_i, v_{-i}) = \pi(v_i', v_{-i}) = \pi(v_i', v_{-T(v) \setminus \{i\}})$ where the second inequality holds by Assumption 3. The argument for a non-trading buyer is similar.   *Q.E.D.*

Hence, the set $T(v) = S(v) \cup B(v)$ of traders may vary as the valuation profile v changes, but the price does not vary for given a set of traders. We note that this constant price may be different for different set of traders, as the following example demonstrates.

Example 4. Suppose there are one seller and two buyers. Let π and π' be two constant prices with $\pi > \pi'$. Let the mechanism be such that

$$(q_1(v), y_1(v)) = \begin{cases} (1, \pi) & \text{if } s_1 < \pi < b_1; \\ (0, 0) & \text{otherwise,} \end{cases}$$

$$(q_2(v), y_2(v)) = \begin{cases} (1, \pi') & \text{if } s_1 < \pi' < b_2 \text{ but not } s_1 < \pi < b_1; \\ (0, 0) & \text{otherwise,} \end{cases}$$

$$(p_1(v), x_1(v)) = (q_1(v) + q_2(v), y_1(v) + y_2(v)).$$

It is straightforward to show that this mechanism satisfies all properties in Proposition 2.

Since both the set of sellers and the set of buyers are finite, the family of the sets of traders is also finite. This leads us to the following definition.

Definition A mechanism is a generalized posted price mechanism if, for each set $T = S \cup B$ of traders with $|S| = |B|$, there is a constant price π such that all sellers in S receive π, all buyers in B pay π, and all other players receive or pay nothing.

Proposition 2 thus states that the price is posted for each set of traders in any robust double auction mechanism that also satisfies Assumptions 1, 2, and 3. Restating it, we have:

Theorem *A robust double auction mechanism that also satisfies Assumption 1 of common price, Assumption 2 of zero payoff for the worst type and Assumption 3 of non-bossiness-in-payments must be a generalized posted price mechanism.*

We note that the converse of the theorem is not true. To see this, consider Example 4 again but with $\pi < \pi'$. The seller with $s_1 < \pi$ has an incentive to

submit s_1' with $\pi \leq s_1' < \pi'$ and trade with buyer 2 when $\pi < b_1$ and $\pi' < b_2$. Thus, this generalized posted price mechanism is not dominant strategy incentive compatible. We also note that the theorem is a statement about the terms of trade, but not about the selection of traders. There may exist many alternative manners of determining the set $T(v) = S(v) \cup B(v)$ of traders for each possible v, and it appears to be a difficult task to provide a clear characterization.

3. CONCLUSION

We have shown that the generalized posted price mechanisms are the only robust double auction mechanisms under some assumptions, but not all generalized posted price mechanisms are robust. Further characterization on the structure of the generalized posted price mechanisms remains as a future research agenda.

Although some of the results can be extended straightforwardly to the class of probabilistic allocation rules, our main characterization has been established for deterministic allocation rules. Hence, it remains as a future research agenda to extend the analysis to probabilistic robust double auction mechanisms.

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