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Optimal Quality Scores in Sponsored Search Auctions: Full Extraction of Advertisers' Surplus*

Kiho Yoon

Abstract

This paper shows that the quality scores in sponsored search auctions can be optimally chosen to extract all the advertisers' surplus. The reason for the full extraction result is that the quality scores may effectively set all the bidders' valuations equal to the highest valuation, which induces intense bidding competition.

KEYWORDS: online advertising, sponsored search, quality score, full extraction

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1 Introduction

The sponsored search auction is an innovative trading institution in online advertising. Whenever an Internet user types in a particular search keyword, a new auction is triggered for advertising slots that will display sponsored search results or ads alongside with the organic search results.[1] Advertisers pay to the advertising intermediary only when their ads are clicked. This trading institution has evolved over time since its inception in 1997.[2] In the current auction format, several advertising slots or positions are simultaneously auctioned off using a payment scheme apparently similar to the second price auction. In particular, each advertiser or bidder pays not his/her own bid but the minimum price that would retain the current position. Edelman *et al.* (2007) call the auction rule in practice as the generalized second-price (GSP) auction.

One of the salient features of this auction is the use of quality scores, which influence the advertisers' positions as well as the minimum bid requirements. As for the positions, the advertisers are not ordered by their bid amounts but by the adjusted bids multiplied by the quality scores. Google initially used the click-through rate to determine the quality score. It later switched to a less transparent system that incorporates such factors as the relevance of the keywords to its ad group, the landing page quality, the advertisers' historical performance, and other relevant factors.[3] Yahoo! initially used only the bids to determine the order, but began to use a ranking system similar to Google's in 2007.

The quality score is designed to ensure that the most relevant ads are shown on the advertising slots. This will generate as many actual clicks as possible, and may help the advertisers and the Internet users as well as Google and Yahoo! The quality score can, in fact, achieve more: This paper shows that, by optimally choosing the quality scores, it is possible to extract all the advertisers' surplus. The reason for the full extraction result is that the quality scores may effectively make advertisers' valuations equal to the highest valuation, thus inducing fierce bidding competition. We note that this result is obtained in a complete information framework, so it may seem obvious in that the advertising intermediary may directly charge advertisers' valuations. We only mention here that we are interested not in proposing conceivable mechanisms, but in analyzing the prevailing trading institution of the

[1] The auction for keywords has started in search advertising, and expanded to contextual advertising on content pages. Though this paper concentrates on the sponsored search auctions, the analysis applies to the wider auctions for keywords.

[2] For the early history of sponsored search advertising, see Battelle (2005) as well as the papers cited below.

[3] The exact formula is not released publicly.

GSP auction with a special attention to the quality score. More discussion on this and other modeling choices are relegated to the last section.

The sponsored search auction has recently attracted much academic attention, especially in computer science. The basic properties of sponsored search auctions have been investigated in early papers including Aggarwal *et al.* (2006), Edelman *et al.* (2007), and Varian (2007). The actual practice and evolution of search advertising is nicely presented in Edelman *et al.* (2007), Evans (2008, 2009), and Liu *et al.* (2009). Other notable papers in economics include Athey and Ellison (2007), Börgers *et al.* (2007), and Milgrom (2009).

2 Main Results

Consider a search advertising market organized by a search site (such as Google), a portal (such as Yahoo!), or any entity that acts as an advertising intermediary. We will henceforth call this entity as the auctioneer since auctions have been used in practice. There are K positions (advertising slots) and I bidders (advertisers) with $K \leq I$. Let c^k be the (expected) number of views that an advertisement in position $k = 1, \ldots, K$ effectively receives. Without loss of generality, order the positions so that $c^1 > \cdots > c^K > 0$. We may also set $c^{K+1} = \cdots = c^I = 0$ for analytic convenience. Each bidder is characterized by two parameters. For $i = 1, \ldots, I$, let r_i be the positive click-through rate (CTR for short) or the rate of clicks when viewed, and let v_i be the positive valuation per click. The CTR is related to the relevance of the advertisement with respect to the particular keyword, while the valuation is related to the final payoff resulting from the clicks. Hence, if bidder i is assigned to position k, the (expected) number of clicks is $c^k r_i$ and the total payoff that bidder i obtains is $c^k r_i v_i$. Note that (i) each position's number of views is independent of the bidder, and (ii) each bidder's valuation is independent of the position.[4]

We describe the auction rule used in search advertising markets, which is termed as the generalized second-price auction by Edelman *et al.* (2007), with an emphasis to the quality score. Consider a specific keyword. Bidders submit nonnegative bids b_i's per click. Bid b_i is multiplied by the quality score $q_i > 0$, and these adjusted bids are arranged in a decreasing order.[5] For this, let $\pi : I \to I$ be the permutation of bidders according to the order of adjusted bids $q_i b_i$ so that $\pi(k)$ is the

[4]In a more general setting, the total payoff of bidder i who is assigned to position k may be defined as $c_i^k v_i^k$. The value v_i^k may decrease in k if the conversion rate (that is, the rate of transactions/actions when clicked) is declining. The case when $c_i^k = c^k r_i$, as we specify here, is known as the separable or multiplicative form. Most papers including Edelman *et al.* (2007) and Varian (2007) adopt the separable form, while Börgers *et al.* (2007) discuss the general setting as well.

[5]Ties can be broken in any pre-specified way.

bidder with the k-th highest adjusted bid. Then, we have $q_{\pi(1)}b_{\pi(1)} \geq \cdots \geq q_{\pi(I)}b_{\pi(I)}$. Bidder $\pi(k)$ for $k = 1, \ldots, K$ is assigned to position k, that is, a bidder with the highest adjusted bid is assigned to the highest position (position 1), a bidder with the second highest adjusted bid is assigned to the second highest position (position 2) and so on. Bidder $\pi(k)$ for $k = 1, \ldots, K$ is charged the minimum price to retain the current position, so pays $q_{\pi(k+1)}b_{\pi(k+1)}/q_{\pi(k)}$ per click. Note that $b_{\pi(K+1)}$ is well-defined when $K < I$. Otherwise, i.e., when $K = I$, we can let $b_{\pi(K+1)} = 0$.[6] A bidder in position k has a net payoff or surplus of $c^k r_{\pi(k)}(v_{\pi(k)} - q_{\pi(k+1)}b_{\pi(k+1)}/q_{\pi(k)})$ for $k = 1, \ldots, K$. A bidder without a position does not pay and has a surplus of zero.

When $q_i = 1$ for all i, the bidders are arranged according to their submitted bids and the winning bidders pay the next bid, i.e., bidder $\pi(k)$ pays $b_{\pi(k+1)}$ per click for $k = 1, \ldots, K$. This corresponds to the original auction format that Yahoo! has used. On the other hand, when $q_i = r_i$ for all i, bidders are arranged according to the CTR-adjusted bids and bidder $\pi(k)$'s total payment is $c^k r_{\pi(k+1)}b_{\pi(k+1)}$. This corresponds to the original auction format that Google has used.

Following Edelman *et al.* (2007) and Varian (2007), we study the static environment with complete information. One of the reasons is that it is extremely complicated, if not impossible, to analyze the generalized second price auction with multiple positions as a game of incomplete information. Moreover, as these papers claim, the assumption of complete information is a reasonable first approximation since all relevant information about bidders is likely to be inferred over time due to the ease of experimenting with bidding strategies in real-world sponsored search auctions.[7]

Definition 1 A Nash equilibrium is a set of bids $\{b_1, \ldots, b_I\}$ that satisfies

$$c^k r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(k+1)}b_{\pi(k+1)}}{q_{\pi(k)}}\Big) \geq c^j r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(j+1)}b_{\pi(j+1)}}{q_{\pi(k)}}\Big) \text{ for } j > k, \text{ and}$$
$$c^k r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(k+1)}b_{\pi(k+1)}}{q_{\pi(k)}}\Big) \geq c^j r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(j)}b_{\pi(j)}}{q_{\pi(k)}}\Big) \text{ for } j < k.$$

[6] Alternatively, we can set $b_{I+1} = 0$ by convention. Since we have set $c^{K+1} = \cdots = c^I = 0$, either convention will do for the following analysis.

[7] Varian (2007, p. 1175) notes, 'It is very easy to experiment with bidding strategies in real-world ad auctions. Google reports click and impression data on an hour-by-hour basis (...) The availability of such tools and services, along with the ease of experimentation, suggest that the full-information assumption is a reasonable first approximation. As we will see below, the Nash equilibrium model seems to fit the observed choices well.' Edelman *et al.* (2007 p. 249) also notes, '... advertisers are likely to learn all relevant information about other's values.'

Hence, bidders do not have incentives to change their assigned positions. Note that this definition reflects the asymmetry: Moving to a higher position requires beating the adjusted bid of who occupies that position, while moving to a lower position requires beating the adjusted bid of who occupies the position next to that position (i.e., the price the bidder of that position pays). A refinement of the Nash equilibrium concept is proven to be extremely useful: Edelman *et al.* (2007) call it the locally envy-free equilibrium and Varian (2007) calls it the symmetric Nash equilibrium.

Definition 2 A symmetric Nash equilibrium (SNE) is a set of bids $\{b_1, \ldots, b_I\}$ that satisfies

$$c^k r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(k+1)}b_{\pi(k+1)}}{q_{\pi(k)}}\Big) \geq c^j r_{\pi(k)}\Big(v_{\pi(k)} - \frac{q_{\pi(j+1)}b_{\pi(j+1)}}{q_{\pi(k)}}\Big) \text{ for } k, j = 1, \ldots I.$$

Equivalently, an SNE set of bids satisfies

$$c^k\Big(q_{\pi(k)}v_{\pi(k)} - q_{\pi(k+1)}b_{\pi(k+1)}\Big) \geq c^j\Big(q_{\pi(k)}v_{\pi(k)} - q_{\pi(j+1)}b_{\pi(j+1)}\Big) \text{ for } k, j = 1, \ldots I.$$

Observe that this definition of SNE gives the inequalities

$$c^k\Big(q_{\pi(k)}v_{\pi(k)} - q_{\pi(k+1)}b_{\pi(k+1)}\Big) \geq c^{k+1}\Big(q_{\pi(k)}v_{\pi(k)} - q_{\pi(k+2)}b_{\pi(k+2)}\Big) \text{ and}$$
$$c^{k+1}\Big(q_{\pi(k+1)}v_{\pi(k+1)} - q_{\pi(k+2)}b_{\pi(k+2)}\Big) \geq c^k\Big(q_{\pi(k+1)}v_{\pi(k+1)} - q_{\pi(k+1)}b_{\pi(k+1)}\Big),$$

which can be combined to get

$$(c^k - c^{k+1})q_{\pi(k+1)}v_{\pi(k+1)} + c^{k+1}q_{\pi(k+2)}b_{\pi(k+2)} \leq c^k q_{\pi(k+1)}b_{\pi(k+1)}$$
$$\leq (c^k - c^{k+1})q_{\pi(k)}v_{\pi(k)} + c^{k+1}q_{\pi(k+2)}b_{\pi(k+2)}.$$

Thus, each bidder's $c^k q_{\pi(k+1)}b_{\pi(k+1)}$ is bounded below and above. We can express the upper and lower boundary cases in the previous inequalities recursively as

$$c^k q_{\pi(k+1)}b^U_{\pi(k+1)} = (c^k - c^{k+1})q_{\pi(k)}v_{\pi(k)} + c^{k+1}q_{\pi(k+2)}b^U_{\pi(k+2)} \text{ and}$$
$$c^k q_{\pi(k+1)}b^L_{\pi(k+1)} = (c^k - c^{k+1})q_{\pi(k+1)}v_{\pi(k+1)} + c^{k+1}q_{\pi(k+2)}b^L_{\pi(k+2)},$$

from which we have

$$c^k q_{\pi(k+1)}b^U_{\pi(k+1)} = \sum_{j=k}^{K}(c^j - c^{j+1})q_{\pi(j)}v_{\pi(j)},$$

$$c^k q_{\pi(k+1)}b^L_{\pi(k+1)} = \sum_{j=k}^{K}(c^j - c^{j+1})q_{\pi(j+1)}v_{\pi(j+1)}.$$

Therefore, the lower bound of the auctioneer's revenue can be expressed in terms of valuations as

$$\sum_{k=1}^{K} c^k r_{\pi(k)} q_{\pi(k+1)} b^L_{\pi(k+1)} / q_{\pi(k)} = \sum_{k=1}^{K} \frac{r_{\pi(k)}}{q_{\pi(k)}} \sum_{j=k}^{K} (c^j - c^{j+1}) q_{\pi(j+1)} v_{\pi(j+1)}$$

and the upper bound can be similarly expressed. We will work with the lower bound. This bound is prominent in that it coincides with the Vickrey payment when the quality score q_i is set to the click-through rate r_i.[8] Moreover, as we show in Proposition 2 below, the optimal revenue even with the lower bound SNE extracts all the bidders' surplus, hence all the other SNEs *a fortiori* achieve the full extraction of surplus.

To keep notations simple, we will henceforth let $r_1 v_1 \geq r_2 v_2 \geq \cdots \geq r_I v_I$ without loss of generality. That is, we rename the bidders in the order of CTR-adjusted valuations $r_i v_i$'s. When the number of bidders exceeds the number of positions, we have:

Proposition 1 *Assume $K < I$. For any $\alpha \in [0, 1]$, the auctioneer can choose the quality scores so that, for all $k = 1, \cdots, K$, the lower bound of the auctioneer's revenue from position k is exactly 100α percent of the total payoff $c^k r_k v_k$ of advertiser k who occupies that position.*

Proof. It will be confirmed shortly that $\pi(k) = k$ all $k = 1, \ldots, K+1$ with our choice of quality scores. Hence, the revenue from position K is $\frac{r_K}{q_K}(c^K - c^{K+1}) q_{K+1} v_{K+1}$. By choosing $q_{K+1} = \frac{\alpha q_K v_K}{v_{K+1}}$, that is, by setting $q_{K+1} v_{K+1} = \alpha q_K v_K$, it becomes $\alpha c^K r_K v_K$ since $c^{K+1} = 0$. Next, the revenue from position $K-1$ is

$$\begin{aligned} &\frac{r_{K-1}}{q_{K-1}} \Big[(c^{K-1} - c^K) q_K v_K + (c^K - c^{K+1}) q_{K+1} v_{K+1} \Big] \\ &= \frac{r_{K-1}}{q_{K-1}} \Big[(c^{K-1} - c^K) + \alpha (c^K - c^{K+1}) \Big] q_K v_K = \frac{r_{K-1}}{q_{K-1}} \Big[c^{K-1} - (1-\alpha) c^K \Big] q_K v_K. \end{aligned}$$

By choosing $q_K = \frac{\alpha c^{K-1} q_{K-1} v_{K-1}}{[c^{K-1} - (1-\alpha) c^K] v_K}$, that is, by setting $[c^{K-1} - (1-\alpha) c^K] q_K v_K = \alpha c^{K-1} \times q_{K-1} v_{K-1}$, it becomes $\alpha c^{K-1} r_{K-1} v_{K-1}$. By backward induction, the revenue from position $k = 1, \ldots, K$ becomes $\alpha c^k r_k v_k$ by choosing

$$q_{k+1} = \frac{\alpha c^k q_k v_k}{[c^k - (1-\alpha) c^{k+1}] v_{k+1}},$$

[8] Edelman *et al.* (2007) give special attention to the lower bound in their Theorem 1. Varian (2007) also argues that the lower bound is the most plausible outcome.

that is, by setting $[c^k - (1-\alpha)c^{k+1}]q_{k+1}v_{k+1} = \alpha c^k q_k v_k$. Since $\frac{\alpha c^k}{c^k-(1-\alpha)c^{k+1}} \leq 1$ for all $\alpha \in [0,1]$ and $k = 1, \ldots, K$, we have $q_1 v_1 \geq q_2 v_2 \geq \cdots \geq q_{K+1}v_{K+1}$. By choosing q_j for $j = K+2, \ldots, I$ to satisfy $q_{K+1}v_{K+1} \geq q_j v_j$, we conclude that $\pi(k) = k$ for $k = 1, \ldots, K+1$ because it is easy to show that the adjusted valuations $q_i v_i$'s respect the order of adjusted bids $q_i b_i$'s in any SNE. ■

This proposition, in particular, implies that the auctioneer can extract all the advertisers' surplus by setting $\alpha = 1$.

Proposition 2 *(Full extraction of advertisers' surplus) Assume $K < I$. The optimal quality scores $(q_1^*, \ldots, q_I^*)$ that maximize the lower bound of the auctioneer's revenue satisfy*

$$q_1^* v_1 = q_2^* v_2 = \cdots = q_{K+1}^* v_{K+1} \geq q_j^* v_j \quad \text{for all} \quad j = K+2, \ldots, I$$

and the auctioneers' optimal revenue is equal to the maximal sum of advertisers' total payoffs, $\sum_{k=1}^{K} c^k r_k v_k$.[9] [10]

Hence, the auctioneer can extract all the advertisers' surplus in the lower bound SNE. Since bidders enjoy a nonnegative surplus in any SNE, this proposition implies that this is also the best outcome for the auctioneer in any SNE.

What is the reason behind these results? It is straightforward to see that, with the optimal quality scores of Proposition 2, we have $b_k = v_k$ for $k = 2, \ldots, K+1$, and the payment per click for bidder k is v_k for $k = 1, \ldots, K$. Thus, the optimal quality scores effectively set all the bidders' valuations equal to the highest valuation, which induces intense bidding competition. In the optimal auction design under incomplete information, Myerson (1981) has shown that the auctioneer can increase revenue by giving bid preferences to weak bidders whose expected willingness to pay is lower.[11] Observe that optimal quality scores work similarly as bid preferences in our complete information setting of the generalized second-price

[9]One obvious choice of the quality scores is to set $q_1^* = 1$, $q_k^* = v_1/v_k$ for $k = 2, \ldots, K+1$, and $q_j^* = 0$ for $j = K+2, \ldots, I$.

[10]Observe that the sum of advertisers' total payoffs is maximized when bidders are placed according to the order of $r_i v_i$'s.

[11]Liu and Chen (2006) exploit this insight in the context of sponsored search auctions. They consider the first-price auction format and only the case of a single position, which are at wide variance with actual practice. See other related papers in their reference, too. On the other hand, Lahaie and Pennock (2007) start discussing quality scores under complete information setting. They move on to the incomplete information setting and, with a specific form of $q_i = r_i^\alpha$ for the quality score, show by simulation that higher correlation between r_i and v_i leads to a smaller optimal α.

auction. Observe also that optimal quality scores not only increase the auctioneer's revenue but in fact extract all the advertisers' surplus.[12]

Notwithstanding the full extraction result, it should be noted that the short-term incentive of exploitation may be checked by the long-term incentive of market cultivation. Simply put, sponsored search auctions cannot survive in the end unless advertisers find their surplus satisfactory. Hence, the advertising intermediaries such as Google and Yahoo! may wish to guarantee normal profits to the advertisers. Proposition 1 above shows that it is possible to extract any portion of advertisers' surplus.

We finally mention that full extraction does not occur when $K = I$ since there cannot be enough competition for positions. A straightforward derivation along the lines of the proof of Proposition 1 shows the following: The revenue from position K is zero, and generally the revenue from position k is $(c^k - c^K)r_k v_k$. Bidder k's surplus is $c^K r_k v_k = c^k r_k v_k - (c^k - c^K)r_k v_k > 0$ for $k = 1, \ldots, K$.

3 Discussion

We have shown that the quality scores can be optimally chosen to extract all (or any portion) of the advertisers' surplus. This proposition was established in the static model of complete information. As we have discussed before Definition 1, this modeling choice is a reasonable first approximation since all relevant information about advertisers are likely to be learned via frequent interactions and experimentations. Nevertheless, the complete information assumption may be problematic if it is hard to identify which of its equilibrium predictions are meaningful approximations. Fortunately in this regard, we have established that all equilibria of the complete information game leads to the same conclusion of full surplus extraction.

On the other hand, the full extraction was shown to be possible only for the symmetric Nash equilibrium outcomes. Comparing Definitions 1 and 2, the symmetric Nash equilibrium replaces the term $q_{\pi(j)}b_{\pi(j)}$ in the Nash equilibrium with the term $q_{\pi(j+1)}b_{\pi(j+1)}$ for positions higher than k-th position. This makes the equilibrium inequalities symmetric both for the higher and the lower positions, and creates a tighter set of restrictions. As Börgers *et al.* (2007) point out, however, this restriction is not justified by usual game-theoretic refinement arguments. They also show that this symmetry restriction significantly reduces the set of equilibrium

[12]We conjecture that full extraction may not be feasible under incomplete information unless valuations are correlated across advertisers in the spirit of Crémer and McLean (1988).

outcomes.[13] Therefore, it remains as an open question to see whether the same result holds for the Nash equilibrium outcomes.[14]

One may argue that the full extraction result is not surprising or even trivial since, under complete information setting, the advertising intermediary can charge each advertiser's true valuation. Observe however that the advertising intermediary is not able to directly charge true valuations, nor bidders pay their own bids, under the established institution of the generalized second price auction. To put it differently, suppose an advertising intermediary gets to know advertisers' true valuations, but it cannot directly charge these valuations due to the trading rule evolved in the business. What we have shown is that the quality score may be used as an effective instrument to charge true valuations in an auction that appears to set lower prices than bidders' stated valuations.

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[13]As a matter of fact, they show in a model without consideration to quality scores, i.e., $q_i = 1$ for all i, that any assignment of positions to bidders may be possible in (asymmetric) Nash equilibria. Varian (2007) observes in a setting with $r_i = q_i = 1$ for all i that, while the upper bound NE revenue coincides with the upper bound SNE revenue, the lower bound NE revenue may be lower than the lower bound SNE revenue.

[14]The symmetry restriction essentially renders the bids as supporting prices, which makes the auction problem amenable to classic assignment problem approach. It might require quite a different approach to derive optimal quality scores and to bound the auctioneer's revenue without this restriction.

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