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Incentive-compatible double auction for Peer-to-Peer energy trading considering heterogeneous power losses and transaction costs

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ABSTRACT

Peer-to-Peer (P2P) energy trading has garnered significant attention due to its potential to enhance energy efficiency and promote decentralized energy systems. Designing an effective P2P energy trading market hinges on making efficient P2P transactions and incentivizing truthful information disclosure among participants. Addressing this challenge, our study introduces a trustful double auction mechanism for P2P energy trading incorporating heterogeneous power losses and transaction costs into its framework. Our mechanism includes efficient matching and allocation rules that deal with the environment in which the amount of power losses and transaction costs vary depending on whom the prosumers trade with by integrating Multi-unit Trade Reduction (MTR) and Vickrey–Clarke–Groves (VCG) mechanisms. Moreover, we present a modified VCG transfer rule that prevents budget deficits in bilateral trading contexts while effectively reducing efficiency loss. It is shown that our mechanism satisfies dominant strategy incentive compatibility, individual rationality, budget balance, and asymptotic efficiency. Numerical analysis validates its performance, highlighting the impacts of power losses and transaction costs on the overall performance of our auction mechanism.

1. Introduction

With the increasing adoption of distributed energy resources (DERs), a new category of energy consumers and producers, known as prosumers, has emerged. The DERs have provided prosumers with opportunities not only to save their electricity bills but also to be able to sell excess power back to the utility grid or neighboring consumers. Under this circumstance, P2P energy trading in which prosumers directly exchange electricity with each other without involving intermediaries has garnered significant attention recently. Because P2P trading can transform the traditional one-way electricity trading system into a decentralized two-way system, it reduces prosumers' reliance on the central grid and improves power reliability. Moreover, it creates opportunities for prosumers to profit from renewable energy sources [1,2].

In the context of P2P energy trading, developing a market clearing mechanism that determines prices and allocations holds essential importance. In designing such a mechanism, it is important to recognize that prosumers have private information on the valuations of power. When the market is cleared by aggregating this information, prosumers may have incentives to misreport their private information to achieve more favorable outcomes strategically. This behavior can significantly reduce the overall efficiency of P2P energy trading. Therefore, it is crucial to design a market mechanism that incentivizes prosumers to reveal their valuations of power truthfully. In other words, reporting their true valuations should be the best response when participating in a market designed under such a mechanism. We refer to a mechanism designed in this way as a "trustful mechanism" [3]. Additionally, it is necessary to encourage voluntary participation in the market. In this regard, the previous approaches to develop P2P energy trading mechanisms such as optimization [4], game theory [5], and blockchain-based frameworks [6] may not be suitable because these methodologies struggle to manage issues related to strategic behavior. In contrast, auctions, which are based on mechanism design theory, allow the designer to set rules that achieve desired outcomes without knowing participants' private information. Therefore, recent research has been actively exploring double auction models and this study is in line with those efforts [7,8].

The P2P energy trading double auction must be designed efficiently to foster long-term active participation from prosumers, with particular attention to minimizing power losses and transaction costs. P2P transactions inherently incur power losses, increasing with longer transmission distances [9,10]. Moreover, trading power over a grid incurs various costs including network investments, operational expenses,

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maintenance costs, and additional costs from platform utilization, which can be referred to as transaction costs collectively.

Unlike traditional electricity systems, P2P energy trading takes place within a decentralized framework, requiring prosumers to be matched with counterpart prosumers for transactions. Since power losses and transaction costs incurred by prosumers vary depending on the matching outcomes and allocations, efficiently matching prosumers and allocating trade volumes can enhance prosumers' utilities by reducing overall power losses and transaction costs. Thus, achieving socially efficient outcomes in auctions for P2P energy trading necessitates considering not only bid prices but also power losses and transaction costs within the auction mechanism. However, existing research on P2P energy trading auctions often fails to consider these factors inside the auction frameworks or focuses narrowly on minimizing the total electrical distances incurred by trades [9,11–13]. Such auction mechanisms determine the outcomes of P2P trading without proper management of power losses and transaction costs, potentially resulting in a significant reduction in social welfare.

In this study, we propose a novel truthful double auction mechanism for P2P energy trading that efficiently matches prosumers and allocates trading volumes by incorporating heterogeneous power losses and transaction costs into the auction mechanism. The auction model is based on the concepts of the Multi-unit Trade Reduction (MTR) mechanism [14,15] and the Vickrey–Clarke–Groves (VCG) mechanism, satisfying dominant strategy incentive compatibility (DSIC), individual rationality (IR), budget balance (BB), and asymptotic efficiency (AsE) properties.

The main contributions of this study are as follows:

- First, we develop efficient matching and allocation rules of the P2P energy trading double auction mechanism in which power loss rates and transaction costs vary depending on whom the prosumers trade with.
- Second, we propose an effective double auction mechanism that is robust to the strategic bidding behaviors, prevents budget deficits, and ensures substantial social welfare by integrating the MTR and VCG mechanisms. Unlike the previous study that combined MTR and VCG, it can effectively address scenarios where both buyers and sellers pay transaction costs while achieving even greater social welfare.
- Lastly, we rigorously demonstrate that the proposed mechanism satisfies the required properties in mechanism design even when power losses and transaction costs are fully incorporated into the auction mechanism.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature to identify the position of our study. Section 3 introduces the auction environment and presents our MTR-VCG double auction mechanism considering power losses and transaction costs. Section 4 describes the results of the numerical analysis. Finally, Section 5 concludes the paper. We present all proofs in Appendix.

2. Literature review

In this study, we design the P2P energy trading double auction mechanism. We present an overview of three streams of literature related to this study: (1) market clearing mechanisms for P2P energy trading, (2) double auction models for P2P energy trading considering power losses and transaction costs, and (3) auction mechanism design with transaction costs.

The first stream of literature related to our work focuses on market clearing mechanisms for P2P energy trading. These can be categorized into three main groups: optimization, game theory, and auction theory. Constrained optimization techniques typically frame the problem as social welfare maximization, utilizing methods like the Alternating Direction Method of Multipliers (ADMM) [16–19]. While effective for achieving market efficiency, these models often neglect agent behavior and interaction. Game-theoretic models, such as Stackelberg game models [1,20], non-cooperative game models [21–23], and coalitional game models [24], attempt to address this gap but our objective is to go beyond modeling agents' interactions and design the auction mechanism that determines appropriate prices and allocations during P2P trading. To reflect individual preferences and ensure socially efficient outcomes, the auction mechanism, particularly the double auction, emerges as a suitable choice for market clearing mechanisms of P2P energy trading. [25] introduced a strategy-proof online double auction for smart grid systems, building upon [14]. Extending this work, [7,8,26] expanded double auction models to more realistic scenarios. [7] proposed a two-stage mechanism for agents participating in day-ahead and real-time markets. Meanwhile, [26] devised an iterative uniform-price auction for trading excess photovoltaic (PV) energy. Despite their advancements, these studies fall short in suitability for P2P energy trading due to neglecting crucial factors such as power losses and transaction costs.

The second stream of literature has explored double auction designs for P2P energy trading, trying to consider network utilization costs and power losses in various ways. The previous approaches introduced continuous double auction models (CDA) [9,11,13], multi-round auction models in active distribution networks [12], and an iterative auction model accounting for prosumers' reputation and distance [10]. [9] presented a decentralized mechanism using a CDA model, specifically investigating P2P energy trading driven by the electrical distance between prosumers. Similarly, [11] proposed two CDA mechanisms that consider the dynamic network fees and safe power supply capability. [13] also proposed a CDA-based P2P platform by using an electrical distance formulation for the utilization fee to elaborate on the effect of the transaction fee on social welfare. On the other hand, a multi-round double auction that integrates the costs of voltage regulation and power loss was proposed in [12]. However, these studies fail to ensure efficient P2P energy trading outcomes as they only reflect network utilization fees and the costs of power losses in prosumers' transfers after matching and allocation decisions are made. In contrast, the authors in [10] introduced a distributed reputation-distance-driven iterative auction mechanism. In this approach, a reputation-distance index is incorporated into the auction matching process to reduce power loss, but transaction costs are not considered. In addition to these studies, various other research in P2P energy trading struggle to consider transaction costs and power losses [27–29], but attempts to fully integrate these factors into auction mechanisms to enhance overall auction efficiency remain limited.

The third stream of literature focuses on auction mechanism designs incorporating transaction costs. [30] introduced a strategy-proof direct mechanism for two-sided exchanges with pair-related transaction costs, focusing on scenarios where agents trade a single unit of a single item. Building upon this, [31] expanded to exchange markets where buyers purchase bundles of goods and sellers sell one unit of a good, accommodating heterogeneous transaction costs. [32] devised a double auction for a bilateral electricity market with power transmission costs and presented a uniform price mechanism. Meanwhile, [33] proposed a modified VCG pricing rule for P2P ridesharing systems. However, it was not budget-balanced. As our study's foundation, [34] designed a double auction for bilateral trade markets in transport service procurement, integrating the MTR and VCG mechanisms to achieve budget balance while addressing heterogeneous transaction costs paid by buyers to sellers.

Inspired by [34], our study employs the MTR and VCG mechanisms while maintaining budget balance and extends their approach to P2P energy trading markets. However, our essential differences lie in the following aspects. First, we explicitly incorporate power loss costs, in addition to transaction costs, into our mechanism to accurately reflect the physical characteristics inherent in P2P energy trading. Second, we address the scenario in which both buyers and sellers pay transaction costs to the P2P energy trading platform. Finally, we propose improved allocation and transfer rules, and we theoretically and empirically demonstrate that our auction mechanism achieves greater social welfare while satisfying DSIC, IR, BB, and AsE.

Table 1
Table of notation.

Notation	Definition
B, S	The set of buyers and the set of sellers
b_i, a_j	The valuations of buyer i and seller j for 1 kWh of power
x_i, y_j	Buyer i's demand and seller j's supply
$\hat{b}_i, \hat{a}_j$	The bid of buyer i and the ask of seller j for 1 kWh of power
l_{ij}	Power losses for trading 1 kWh of power between buyer i and seller j
t_{ij}	Transaction costs for trading 1 kWh of power between buyer i and seller j
δ	Coefficient capturing the relationship between the electrical distance and power losses
γ	Coefficient capturing the relationship between the electrical distance and transaction costs
d_{ij}	Electrical distance between buyer i and seller j
$V(B,S)^{-i}$	Optimal objective value of $P(B,S)$ after removal of buyer i
$V(B,S)^{-j}$	Optimal objective value of $P(B,S)$ after removal of seller j
B_R, S_R	Revised set of buyers and sellers
$\tilde{B}, \tilde{S}$	The set of the winning buyers and the winning sellers
$SW(B,S)$	Social welfare corresponding to the buyer set B and the seller set S
u_i, u_j	Buyer i's utility and seller j's utility
p^m	Threshold price
q_{ij}	Trading volume between buyer i and seller j
$\tilde{p}_i, \tilde{p}_j$	Transfer of the winning buyer i and the winning seller j

3. Model

3.1. Auction environment

The present study considers the P2P energy trading market within a local community, operated by a platform. In this local community, we assume that there is a sufficient number of potential sellers and buyers including prosumers with diverse DERs, referred to as agents. Connected to the main power grid, the local community facilitates energy exchange and communication among agents through a power and information network.

Now suppose that the platform would operate the market using a double auction mechanism. Basically, it is assumed that the retail price set by the main grid exceeds the price at which PV energy can be sold to the grid. Agents are motivated to engage in the P2P energy trading market to secure more advantageous deals than those available through the main grid. Their goal is to maximize profits by selling at a higher price or purchasing at a lower price within the P2P market. Agents, who may become either a buyer or a seller depending on the state of their power generation and consumption, participate in P2P trading through the double auction mechanism. Following the auction rule, any unmet demand or surplus supply from prosumers that would not been traded is settled with the main grid at a predetermined rate. We list the notations used throughout the paper in Table 1.

Let B denote the set of buyers and S denote the set of sellers. Now suppose that a buyer $i \in B$ whose true valuation for 1 kWh is b_i wants to buy x_i kWh of electricity, and a seller $j \in S$ whose true valuation for 1 kWh is a_j wants to sell y_j kWh. The true valuations of buyers and sellers follow independent and identically distributed continuous distributions $F(b_i)_{i\in B}$ and $G(a_j)_{j\in S}$, correspondingly, with the respective supports $[\underline{b}, \overline{b}]$ and $[\underline{a}, \overline{a}]$. We assume that agents' valuations are private information, but the distribution functions are public information. Generally, the power consumption and generation levels of prosumers are measured and recorded in real-time through smart meters and generation meters, and this data is linked to the P2P platform, making it observable. Therefore, we assume that agents' demands and supplies are public information. In our double auction, agents bid on two aspects: the desired transaction price and the desired transaction volume. Buyer i bids in the form of (bid ($\hat{b}_i$), demand (x_i)), and seller j bids in the form of (ask ($\hat{a}_j$), supply (y_j)). Agents with the same bid or ask can be uniquely ordered using factors such as their trading history.

When buyer i and seller j trade 1 kWh of power, it is assumed that there is a power loss of l_{ij} kWh, and the corresponding buyer and seller must pay a total transaction cost of t_{ij} to the platform. The values of l_{ij} and t_{ij} are determined according to the electrical distance and coefficients depending on the physical characteristics of the network and the platform's decision. The magnitude of power losses and transaction costs is also contingent upon the traded power quantity. Since calculating the electrical distance d_{ij} between buyer i and seller j is not within our research scope, we assume that the Thevenin impedance distance method [9,27] allows its calculation for all buyer–seller pairs.

If buyer i purchases $\sum_{j\in S} q_{ij}$ from sellers, the amount of power losses corresponding to buyer i's transactions is calculated as follows:

$$PL_i = \sum_{j\in S} l_{ij} q_{ij}, \tag{1}$$

where $l_{ij} = \delta d_{ij}$. The coefficient δ represents the losses factor, determined based on the physical characteristics of the power network, capturing the relationship between the electrical distance and the power losses.

For buyer i, the total transaction cost associated with using the network and the P2P trading platform is as follows:

$$T_i = \sum_{j\in S} \alpha t_{ij} q_{ij}, \tag{2}$$

where $t_{ij} = \gamma d_{ij}$ [9,28]. We assume that l_{ij} and t_{ij} for every buyer $i \in B$ and seller $j \in S$ are public information. The value of $\alpha \in [0,1]$ represents the rate at which the buyer contributes to the transaction cost for a specific transaction. For simplicity, we assume that the transaction costs paid to the platform are shared equally between the buyer and the seller involved in the corresponding transaction. That is, we assume α to be equal to $\frac{1}{2}$. The coefficient γ is determined by the platform, capturing the relationship between the electrical distance and the transaction costs.

3.2. MTR and VCG mechanisms

In this section, we introduce the MTR and VCG mechanisms, which form the basis of our auction mechanism. [14] proposed a trade reduction mechanism that is DSIC and AsE in a multi-unit exchange scenario for a single type of product. Under the trade reduction mechanism, the least profitable trade is excluded from the social welfare-maximizing allocation, but the least profitable bidding prices determine the uniform transaction prices. In contrast, the VCG mechanism clears the market by achieving an efficient allocation that maximizes social welfare and determines transfers based on the marginal contribution to the increase in social welfare [35]. The VCG mechanism is also DSIC and, unlike MTR, is allocative efficient (AE). However, it leads to a budget deficit in bilateral trading situations. We provide detailed definitions of the two auction mechanisms in Appendix A.

3.3. MTR-VCG Double Auction (MVDA) mechanism considering power losses and transaction costs

In this section, we propose a novel trustful P2P energy trading double auction by elaborating on the MVDA mechanism. Specifically, we present the process of determining auction winners, allocating transaction volumes, and computing payments and revenues. Furthermore, we aim to demonstrate whether our proposed mechanism satisfies the primary goals of mechanism design: (1) *DSIC* - truthful bidding is a (weakly) dominant strategy equilibrium, (2) *IR* - every agent in the market participates in the auction voluntarily since all agents have non-negative utility from participation, (3) *BB* - the trade does not run at a deficit, and (4) *AsE* - the market inefficiency under the mechanism compared to the maximal social welfare converges to zero as the number of participants approaches infinity [35].

First, suppose that all agents have submitted their bids. We assume that agents have quasi-linear utility functions and the outside payoffs that agents can obtain when they do not participate in P2P trading are normalized to zero. Then, the maximum achievable social welfare, assuming truthfulness in those bids, is determined by solving the following linear programming problem.

$$P(B,S): \quad \max \quad V(B,S) = \sum_{i\in B}\sum_{j\in S} q_{ij}\{(1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} - \hat{a}_j - \frac{1}{2}t_{ij}\} \tag{3}$$

$$\text{s.t.} \quad \sum_{j\in S} q_{ij}(1-l_{ij}) \le x_i, \quad \forall i \in B \tag{4}$$

$$\sum_{i\in B} q_{ij} \le y_j, \quad \forall j \in S \tag{5}$$

$$q_{ij} \ge 0, \quad \forall i \in B,\ j \in S. \tag{6}$$

Condition (4) represents that a buyer cannot purchase more than the desired quantity and condition (5) ensures that a seller cannot sell more than the available quantity. Ultimately, $P(B,S)$ implies that if all agents bid honestly, we can induce efficient P2P transactions that maximize social welfare by solving this problem.

After determining the optimal solution q^*_{ij} from $P(B,S)$, we establish the threshold price for agents. The threshold price is a benchmark to identify pairs of buyers and sellers that enhance social welfare. The threshold price p^m is computed using $\hat{tb}$ and $\hat{ta}$ as (9). To achieve fairness between buyers and sellers and simplify the model, we set p^m as a middle value of $\hat{tb}$ and $\hat{ta}$.

$$\hat{tb} = min\{(1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} | q^*_{ij} > 0, i \in B, j \in S\} \tag{7}$$

$$\hat{ta} = max\{\hat{a}_j + \frac{1}{2}t_{ij} | q^*_{ij} > 0, i \in B, j \in S\} \tag{8}$$

$$p^m = \frac{\hat{tb} + \hat{ta}}{2} \tag{9}$$

The actual value of the bid (ask) can vary depending on the seller (buyer) with whom the trade occurs. Therefore, in determining the threshold price, we introduce the total bid (ask) concept, which varies depending on the matched counterpart. The expression $(1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij}$ represents the total bid per unit made by buyer i to seller j, considering both power losses and transaction costs incurred during the trade of q^*_{ij}. This value considers the buyer receiving only $q^*_{ij}(1-l_{ij})$ due to power losses and paying half of the transaction costs t_{ij} when a bilateral trading contract for q^*_{ij} is established. On the seller's side, $\hat{a}_j + \frac{1}{2}t_{ij}$ denotes the total ask made by seller j to buyer i, considering transaction costs incurred during the trade of q^*_{ij}.

$\hat{tb}$ corresponds to the lowest total bid value for trading with sellers in the set S under the optimal solution of $P(B,S)$. Similarly, $\hat{ta}$ represents the highest total ask value for trading with buyers in the set B. Because the power loss rates and transaction costs are heterogeneous for each matching between agents, the lowest total bid $\hat{tb}$ may be less than the highest total ask $\hat{ta}$. Consequently, two distinct cases may occur: (1) $\hat{tb} \le \hat{ta}$ or (2) $\hat{tb} > \hat{ta}$. To utilize the threshold price p^m in the transfer rule while ensuring the IC constraint, we adapt the MTR mechanism differently for each scenario.

(1) $\hat{tb} \le \hat{ta}$ (*i.e.* $\hat{tb} \le p^m \le \hat{ta}$)

If the scenario aligns with Case 1, using the threshold price p^m, we exclude some trading relationships by updating the parameters. Initially, for each buyer i, we eliminate any trading relationship with seller j where $(1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij}$ does not surpass p^m by setting the corresponding l_{ij} to 1. Note that specific $i-j$ relationships are eliminated, not all potential trading opportunities of the buyer i. Analogously, for each seller j, any trading relationship with buyer i where $\hat{a}_j + \frac{1}{2}t_{ij}$ is not less than p^m are excluded by adjusting the corresponding l_{ij} to 1.

(2) $\hat{tb} > \hat{ta}$ (*i.e.* $\hat{ta} < p^m < \hat{tb}$)

In Case 2, we follow a similar procedure as described above. Additionally, we set the corresponding l_{ij} to 1 to eliminate the $i-j$ pair that generates $\hat{tb}$ and $\hat{ta}$. If we were to remove trading relationships based solely on p^m as in Case 1, the trading relationships that determined $\hat{tb}$ and $\hat{ta}$ could remain. Since we aim to use p^m in the transfer rule to satisfy the BB constraint, this additional step helps reduce the incentive for strategic bidding behavior aimed at obtaining a more favorable p^m value.

Following this procedure, the revised sets of buyers and sellers can be identified as follows.

$$B_R = \{i | \exists l_{ij} \ne 1, i \in B, j \in S\} \quad \text{and} \quad S_R = \{j | \exists l_{ij} \ne 1, i \in B, j \in S\} \tag{10}$$

Using the revised sets and updated parameters of l_{ij}, we solve a similar linear programming problem to the previously defined one, denoted as $P(B_R, S_R)$, seeking efficient matchings and allocations. The buyers and sellers engaged in power transactions through the MVDA mechanism are identified as follows, with $\tilde{q}^*_{ij}$ denoting the optimal solution of $P(B_R, S_R)$.

$$\tilde{B} = \{i | \sum_{j\in S_R} \tilde{q}^*_{ij}(1-l_{ij}) > 0, i \in B_R\} \quad \text{and} \quad \tilde{S} = \{j | \sum_{i\in B_R} \tilde{q}^*_{ij} > 0, j \in S_R\} \tag{11}$$

Transaction prices are determined based on the modified VCG mechanism. Since the VCG mechanism is not budget-balanced in the case of bilateral trading, the authors in [34] made slight modifications to a transfer rule to avoid creating a deficit. We further adapt and enhance their transfer rule to suit our P2P energy trading problem, providing an advanced rule to improve social welfare. The transfer rule we propose is as follows.

$$\begin{aligned}\tilde{p}_i = {} & V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1)^{-i} \\ & - \{V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1) \\ & - \sum_{j\in S_R} \tilde{q}^*_{ij}((1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij})\} + \sum_{j\in S_R} \tilde{q}^*_{ij} \cdot \frac{1}{2}t_{ij}\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{p}_j = {} & -V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1)^{-j} \\ & + \{V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1) + \sum_{i\in B_R} \tilde{q}^*_{ij}(\hat{a}_j + \frac{1}{2}t_{ij})\} \\ & - \sum_{i\in B_R} \tilde{q}^*_{ij} \cdot \frac{1}{2}t_{ij}\end{aligned} \tag{13}$$

Both buyers' payments and sellers' revenues are composed of each agent's marginal contribution to the increase in social welfare and the imposed transaction costs. In buyer i's payment, we set the total asks of sellers whose trading relationships are not eliminated, i.e., where l_{ij} is not 1, to p^m. Similarly, when determining seller j's revenue, we set the total bids of buyers whose l_{ij} are not 1 to p^m. By modifying how we calculate marginal contributions in the VCG transfer rule, we can utilize a budget-balanced VCG mechanism even in a bilateral trading environment. Since $\tilde{q}^*_{ij}$ is required in the transfer rule instead of q^*_{ij}, it is necessary to obtain the revised sets B_R and S_R before calculating the

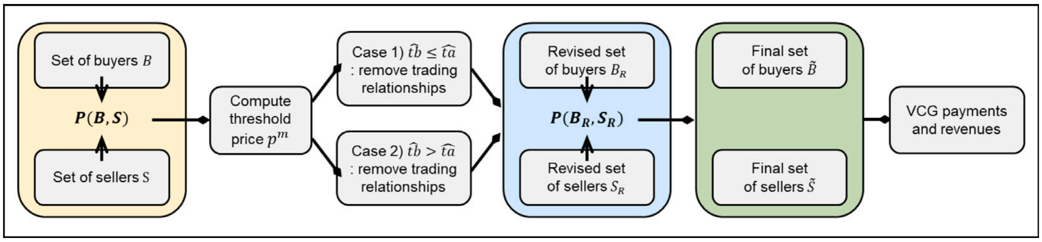

Fig. 1. Procedure of the MVDA mechanism.

transaction prices. The social welfare resulting from the auction can be calculated as follows.

$$SW(B, S) = \sum_{i \in \tilde{B}} u_i + \sum_{j \in \tilde{S}} u_j \tag{14}$$

$$= \sum_{i \in \tilde{B}} \left\{ \sum_{j \in \tilde{S}} \tilde{q}^*_{ij}(1 - l_{ij})b_i - \tilde{p}_i \right\} + \sum_{j \in \tilde{S}} \left\{ \tilde{p}_j - \sum_{i \in \tilde{B}} \tilde{q}^*_{ij} a_j \right\} \tag{15}$$

Finally, Fig. 1 illustrates the summarized auction procedure. It describes the process through which the final winning buyers and winning sellers, who receive the allocation, are determined from the sets of participating buyers and sellers in the auction. The following theorems demonstrate that the MVDA mechanism is DSIC, IR, BB, and AsE.

Theorem 1. *The MVDA mechanism satisfies dominant strategy incentive compatibility, individual rationality, and budget balance.*

All proofs are provided in Appendix A. Theorem 1 shows that agents bid truthfully in equilibrium under the proposed mechanism because there is no incentive to bid values other than their true valuations. Moreover, Theorem 1 asserts that the proposed mechanism ensures agents' non-negative utility and does not bring any deficit to the market. Consequently, agents and the platform find favorable incentives to engage in the auction for P2P energy trading.

Before proving Theorem 2, we assume that the buyers' demand and sellers' supply are both bounded. Furthermore, similar to the assertion in [31], we assume that all agents are located within a compact domain H, which means that the locations of buyers and sellers are in this domain H. The next Theorem 2 shows that the social welfare generated by the MVDA mechanism tends to be maximized as the number of participants increases.

Theorem 2. *The MVDA mechanism is asymptotically efficient.*

The authors in [34] proposed a Modified Multi-unit Trading Reduction and Modified Vickrey–Clarke–Groves (MMTR-MVCG) model for online freight platforms with transaction costs. Similar to our approach, this model is based on the MTR and VCG mechanisms and considers heterogeneous transaction costs within a double auction. Therefore, we use their model as a benchmark to compare the performance of our proposed mechanism. To do so, we have modified their allocation and transfer rules to suit the P2P energy trading problem, detailed in Appendix B. From now on, we will refer to this mechanism as a modified MMTR-MVCG mechanism. The next proposition shows that the MVDA mechanism improves social welfare compared to the modified MMTR-MVCG mechanism.

Proposition 1. *In the MVDA mechanism, buyers and sellers achieve equivalent or greater utility than the modified MMTR-MVCG mechanism.*

The proof is provided in Appendix B. Social welfare improvement stems from two sources. Firstly, fewer valid trading relationships are removed by using the threshold price p^m, allowing more agents to participate in trading. Secondly, when using the modified VCG mechanism on the revised sets of agents obtained in this manner, buyers can purchase at a lower price and sellers can sell at a higher price, even when engaging in transactions of the same quantity.

Table 2
The bidders' information.

Category	Number of bidders	Demand/Supply (kWh-1h)	Valuation (KRW/kWh)
Buyer	20	$N(0.5, 0.05^2)$	$U(120, 150)$
Seller	15	$N(0.7, 0.05^2)$	$U(110, 140)$

4. Numerical analysis

In this section, we analyze the effectiveness of the MVDA mechanism by examining social welfare, budget surplus, trading volume, and the number of participating agents. To provide a comprehensive analysis, we compare the outcomes of the MVDA mechanism with those of the double auction mechanism not considering transaction costs (DA-NCTC), the modified MMTR-MVCG mechanism, and the VCG mechanism.

4.1. Parameter setting

In this numerical analysis, we simulate a scenario with 20 buyers and 15 sellers, each equipped with 3 kWh small-scale solar panels. Power generation and consumption fluctuate based on the trade time, however, we use the following parameters to focus on analyzing the performance of the MVDA mechanism. Buyers' demands and sellers' supplies in an hour follow a normal distribution $N(0.5, 0.05^2)$ and $N(0.7, 0.05^2)$, respectively. This is based on Korea's average residential electricity consumption and average solar power generation data of 3 kWh solar panel, but it has been adjusted for convenience in the experiment [36]. Buyers' true valuations for 1 kWh of power are assumed to follow a uniform distribution $U(120, 150)$, based on the publicly announced residential energy charges in Korea for 2023 [37]. Sellers' true valuations for 1 kWh of power are assumed to follow a uniform distribution $U(110, 140)$, based on the Korean Energy Economics Institute's estimation of the Levelized Cost of Electricity [38]. Table 2 summarizes the parameters and distributions used in the numerical analysis.

For simplicity, the electrical distance d_{ij} is assumed to be uniformly distributed between 1 and 10. To conduct a sensitivity analysis and assess the impact of varying parameters that determine transaction costs and power losses, as defined in (1) and (2), we consider γ ranging from 0.25 to 1.0 with incremental increases of 0.25, and δ ranging from 0.001 to 0.009 with incremental increases of 0.002, i.e., $\gamma \in \{0.25, 0.5, 0.75, 1.0\}$ and $\delta \in \{0.001, 0.003, 0.005, 0.007, 0.009\}$. Despite the limited prevalence of P2P transactions in Korea, we set γ and δ based on Korea's average network utilization fee rate, approximately 4 KRW/kWh, and power loss rate, about 3.5% [36,37,39].

DA-NCTC is a modified MVDA mechanism that matches agents and allocates trading volumes without considering transaction costs. We

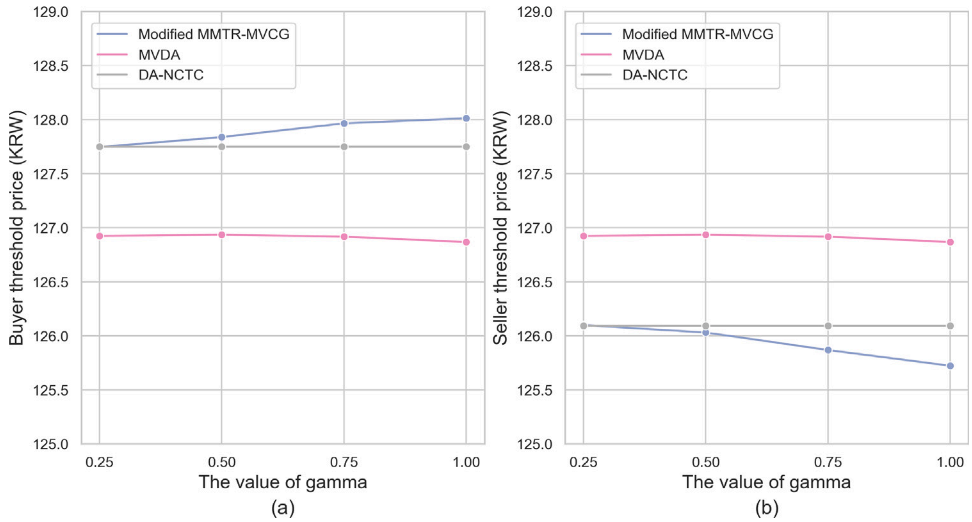

Fig. 2. Comparison of threshold prices.

Table 3
Performance comparison of auction mechanisms across different γ values (KRW, ₩).

Transaction cost rate γ		0.25	0.5	0.75	1.0
Total social welfare	MVDA	168.44	164.49	160.54	156.66
	DA-NCTC	166.20	161.59	156.69	152.07
	Modified MMTR-MVCG	167.81	163.52	159.10	154.67
	VCG	169.28	165.58	161.93	158.33
Social welfare	MVDA	160.20	155.35	150.72	145.86
	DA-NCTC	147.17	141.80	135.97	130.94
	Modified MMTR-MVCG	149.62	143.50	136.74	130.64
	VCG	182.39	180.07	177.72	175.38
Budget surplus	MVDA	8.24	9.14	9.82	10.80
	DA-NCTC	19.03	19.79	20.72	21.13
	Modified MMTR-MVCG	18.19	20.02	22.36	24.03
	VCG	−13.10	−14.49	−15.80	−17.05

emphasize the importance of considering transaction costs in determining agents' matchings and allocations by comparing the performance of the MVDA mechanism with that of the DA-NCTC mechanism. Even in a P2P trading market utilizing the DA-NCTC mechanism, buyers cannot achieve the desired trading volume due to power losses. To address this, power losses are incorporated into the allocation process of the DA-NCTC. However, in the DA-NCTC mechanism, transaction costs are only introduced to transfers after the matching and allocations are determined.

4.2. Results

In our analysis, we assume that auctions are conducted once every hour, and we focus on the results from a single auction. To ensure robustness, the simulation was repeated 1000 times for each γ and δ scenario, with the average results presented. The analysis was performed on a PC with a 3.6 GHz Intel Core i7-7700 CPU and 8 GB RAM under Windows 10. All computations were conducted using Python.

Table 3 presents a comparative analysis of social welfare and budget surplus across various mechanisms, considering different values of transaction cost rate γ. The total social welfare encompasses the utilities of buyers, sellers, and the platform, while the social welfare comprises the utilities of buyers and sellers. The surplus represents the platform's revenue, excluding transaction costs, calculated by deducting trading sellers' revenue and agents' transaction costs from the payment made by trading buyers. Therefore, the total social welfare can be understood as the sum of social welfare and budget surplus, and it can be calculated as $V(B_R, S_R)$ according to (3).

Irrespective of γ values, the total social welfare follows an ascending order: DA-NCTC, modified MMTR-MVCG, MVDA, and VCG. However, the VCG mechanism consistently generates a negative surplus, resulting in a budget deficit. The MVDA mechanism has greater social welfare than the modified MMTR-MVCG mechanism. In addition, the difference in social welfare between the MVDA and the modified MMTR-MVCG mechanisms increases when the transaction cost rate increases. As γ increases, the transaction costs imposed per unit of trade increase. Consequently, both total social welfare and social welfare decrease. Meanwhile, at $\gamma = 1$, the DA-NCTC slightly outperforms the modified MMTR-MVCG in terms of agents' social welfare due to the greater budget surplus provided by the modified MMTR-MVCG mechanism when transaction costs rise.

Fig. 2 shows the impacts of transaction cost rate on the threshold prices in the mechanisms. In the MVDA mechanism, changes in the transaction cost rate do not significantly affect the threshold price. However, in the modified MMTR-MVCG mechanism, the buyers' threshold price increases and the sellers' threshold price decreases as the transaction cost rate increases. In the modified MMTR-MVCG mechanism, even with higher transaction costs, the stricter enforcement of threshold prices on agents leads to increased efficiency losses as the transaction cost rate rises.

From Fig. 3, agents' total transaction costs under three mechanisms increase as the transaction cost rate increases. Notably, the total transaction costs incurred by the MVDA mechanism consistently exceed those of the other mechanisms. This is because our mechanism has a higher transaction rate.

Table 4 presents the impacts of transaction costs on total trading volume and the number of agents who successfully trade under each mechanism. The total trading volume and the number of trading agents decrease as the transaction cost rate increases in every mechanism.

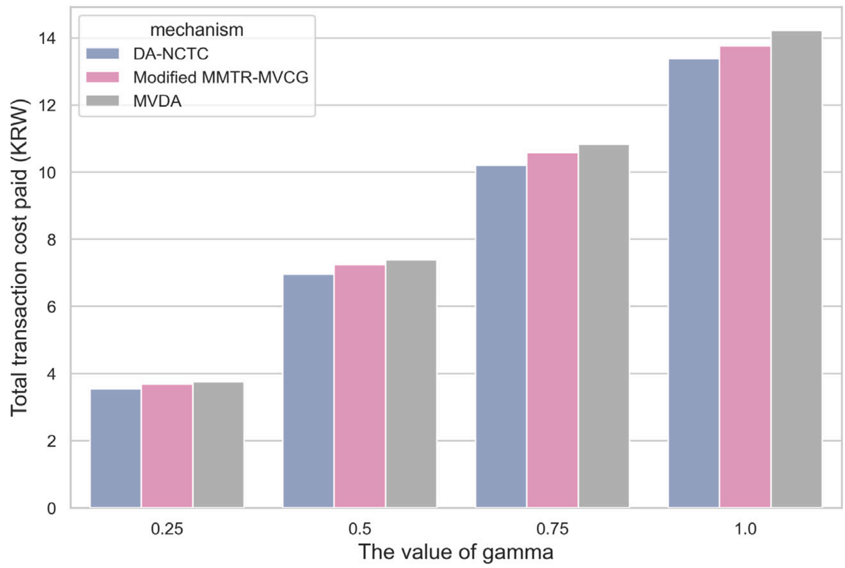

Fig. 3. Comparison of the total transaction costs.

Table 4
Transaction rates comparison of auction mechanisms across different γ values.

Transaction cost rate γ		0.25	0.5	0.75	1.0
Total trading volume (kWh)	MVDA	6.99	6.88	6.76	6.66
	DA-NCTC	6.48	6.32	6.15	6.00
	Modified MMTR-MVCG	6.83	6.67	6.50	6.34
The number of trading buyers	MVDA	14.26	14.05	13.85	13.66
	DA-NCTC	13.07	12.69	12.31	11.98
	Modified MMTR-MVCG	13.87	13.58	13.28	12.95
The number of trading sellers	MVDA	10.23	10.09	9.93	9.80
	DA-NCTC	9.62	9.52	9.44	9.30
	Modified MMTR-MVCG	10.03	9.80	9.59	9.37

However, the MVDA mechanism tends to make more transactions than the other mechanisms irrespective of the transaction cost rate. Thus, our mechanism can attract more potential agents to participate in the P2P energy trading auctions.

Table 5 shows that total social welfare and social welfare under the four mechanisms decrease as the power loss rate increases. The efficiencies realized by the mechanisms when the power loss rate is below 0.007 sorted from the largest to the smallest are as follows: VCG, MVDA, modified MMTR-MVCG, DA-NCTC. On the other hand, the VCG mechanism always generates a significant budget deficit, and the deficit increases as the power loss rate increases. Hence, it is not suitable for the P2P energy trading platform to use the VCG mechanism to operate the market. Thus, the MVDA mechanism can match the agents more efficiently without making any deficit regardless of the power loss rates.

Table 6 presents the impacts of power loss rates on total trading volume and the number of trading agents. Both metrics decrease as the power loss rate increases. Remarkably, the MVDA, the modified MMTR-MVCG, and the DA-NCTC mechanisms consistently outperform each other in that sequence, irrespective of δ.

5. Conclusion

P2P energy trading has emerged as a solution for enhancing power system stability and offering economic benefits to prosumers. The key challenge in a P2P energy trading market is motivating prosumers to truthfully reveal their information while efficiently matching prosumers and allocating trading volumes. To this end, we proposed a truthful double auction mechanism for P2P energy trading by integrating the MTR and VCG mechanisms. Our approach includes efficient matching and allocation rules that account for heterogeneous power losses and transaction costs. We developed an effective auction rule that revises the sets of buyers and sellers using a threshold price and employs a modified VCG transfer rule to avoid budget deficits while ensuring sufficient social welfare. We showed that our double auction mechanism, the MVDA mechanism, is DSIC, IR, BB, and AsE.

Through comparative evaluations under various market scenarios, we assessed the performance of the MVDA mechanism in terms of social welfare, budget surplus, total trading volume, and the number of trading agents. Firstly, the MVDA mechanism exhibits more transactions and greater efficiency compared to a double auction mechanism that does not incorporate transaction costs, and it prevents budget deficits compared to the VCG mechanism, which shows significant deficits. Secondly, unlike the modified MMTR-MVCG mechanism, which results in a large budget surplus, our MVDA mechanism provides greater utility to agents. Lastly, we observed that the number of transactions and trading agents decreases as transaction costs and power loss rates increase.

While this paper proposed an effective approach to designing a truthful double auction to address heterogeneous power losses and transaction costs, several areas for improvement remain. First, we did not consider network constraints when finding efficient allocations,

Table 5
Performances comparison of auction mechanisms across different δ values (KRW, ₩).

Power loss rate δ		0.001	0.003	0.005	0.007	0.009
Total social welfare	MVDA	170.30	166.26	162.10	158.04	153.88
	DA-NCTC	167.72	163.73	159.31	154.92	150.37
	Modified MMTR-MVCG	169.85	165.60	161.08	156.48	151.55
	VCG	170.79	166.97	163.20	159.46	155.76
Social welfare	MVDA	163.74	158.84	153.54	148.30	143.00
	DA-NCTC	148.27	145.49	140.20	135.39	129.81
	Modified MMTR-MVCG	153.83	148.03	141.42	134.57	127.17
	VCG	181.98	179.76	177.53	175.24	172.91
Budget surplus	MVDA	6.56	7.42	8.56	9.74	10.88
	DA-NCTC	19.45	18.24	19.11	19.54	20.56
	Modified MMTR-MVCG	16.02	17.57	19.66	21.91	24.38
	VCG	−11.19	−12.78	−14.33	−15.78	−17.15

Table 6
Transaction rates comparison of auction mechanisms across different δ values.

Power loss rate δ		0.001	0.003	0.005	0.007	0.009
Total trading volume (kWh)	MVDA	7.07	6.95	6.82	6.72	6.62
	DA-NCTC	6.47	6.41	6.27	6.16	6.02
	Modified MMTR-MVCG	6.92	6.78	6.62	6.47	6.29
The number of trading buyers	MVDA	14.47	14.20	13.92	13.68	13.45
	DA-NCTC	13.11	12.93	12.58	12.29	12.00
	Modified MMTR-MVCG	14.11	13.81	13.45	13.12	12.74
The number of trading sellers	MVDA	10.32	10.15	9.99	9.86	9.71
	DA-NCTC	9.61	9.56	9.44	9.37	9.28
	Modified MMTR-MVCG	10.16	9.94	9.74	9.52	9.28

focusing instead on designing the auction mechanism. Future work could explore how to apply our mechanism in markets with various network constraints. Secondly, we assumed that demand and supply information were public information. It is theoretically challenging to design an incentive-compatible double auction mechanism that also treats demand and supply information as private. However, developing a two-dimensional double auction mechanism is crucial for understanding the P2P energy trading market and improving its implementability. Therefore, future research could explore an auction mechanism that accounts for private information on demand and supply.

CRediT authorship contribution statement

Jisu Sim: Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis. **Deok-Joo Lee:** Writing – review & editing, Supervision, Project administration. **Kiho Yoon:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

A.1. MTR and VCG mechanisms

Definition 1 (*Multi-unit Trade Reduction Mechanism*)**.** Order buyers and sellers based on their bid and ask prices, i.e., $\hat{b}_{[1]} \geq \cdots \geq \hat{b}_{[|B|]}$ and $\hat{a}_{[1]} \leq \cdots \leq \hat{a}_{[|S|]}$. Define L and K such that either (A.1) satisfies or (A.2) satisfies.

$$\hat{b}_{[K]} \geq \hat{a}_{[L]} \geq \hat{b}_{[K+1]} \quad \text{and} \quad \sum_{1}^{L-1} y_{[j]} \leq \sum_{1}^{K} x_{[i]} \leq \sum_{1}^{L} y_{[j]} \tag{A.1}$$

$$\hat{a}_{[L+1]} \geq \hat{b}_{[K]} \geq \hat{a}_{[L]} \quad \text{and} \quad \sum_{1}^{K-1} x_{[i]} \leq \sum_{1}^{L} y_{[j]} \leq \sum_{1}^{K} x_{[i]} \tag{A.2}$$

Check whether inequality $\sum_1^{K-1} x_{[i]} \geq \sum_1^{L-1} y_{[j]}$ (overdemand) or $\sum_1^{K-1} x_{[i]} \leq \sum_1^{L-1} y_{[j]}$ (oversupply) holds. Suppose that (A.1) and overdemand inequality hold. Then, the multi-unit trade reduction mechanism with the allocation rule $((a_i^M)_{i\in B}, (a_j^M)_{j\in S})$ and the transfer rule $((p_i^M)_{i\in B}, (p_j^M)_{j\in S})$ is given by

$$a_i^M = \begin{cases} x_i - \frac{\sum_1^{K-1} x_{[i]} - \sum_1^{L-1} y_{[j]}}{K-1}, & \text{if } [i] < [K] \\ 0, & \text{otherwise} \end{cases}, \quad a_j^M = \begin{cases} y_j, & \text{if } [j] < [L] \\ 0, & \text{otherwise} \end{cases} \tag{A.3}$$

$$p_i^M = \begin{cases} \hat{b}_{[K]}, & \text{if } [i] < [K] \\ 0, & \text{otherwise} \end{cases}, \quad p_j^M = \begin{cases} \hat{a}_{[L]}, & \text{if } [j] < [L] \\ 0, & \text{otherwise} \end{cases} \tag{A.4}$$

The transfer rule is always defined as in (A.4), regardless of the case, and the allocation rule can similarly be defined for cases where (A.2) holds or in the case of oversupply. Before moving on to the definition of the VCG mechanism, let us consider the following optimization problem.

$$P^V(B,S): \quad \max \quad G(B,S) = \sum_{i\in B}\sum_{j\in S} q_{ij}(\hat{b}_i - \hat{a}_j) \tag{A.5}$$

$$\text{s.t.} \quad \sum_{j\in S} q_{ij} \le x_i, \quad i \in B \tag{A.6}$$

$$\sum_{i\in B} q_{ij} \le y_j, \quad j \in S \tag{A.7}$$

$$q_{ij} \ge 0, \quad \forall i, j. \tag{A.8}$$

Definition 2 (*Vickrey–Clarke–Groves Mechanism*)**.** The Vickrey–Clarke–Groves mechanism is an efficient mechanism with the allocation rule $(q^V_{ij})_{i\in B,\, j\in S}$ and the transfer rule $((p^V_i)_{i\in B},\, (p^V_j)_{j\in S})$ which are given by

$$q^V_{ij} = q^*_{ij}, \quad \forall i,\ j, \tag{A.9}$$

$$p^V_i = \sum_{j\in S} q^V_{ij}\hat{b}_i - (G(B,S) - G(B,S)^{-i}), \quad \forall i, \tag{A.10}$$

$$p^V_j = \sum_{i\in B} q^V_{ij}\hat{a}_j + (G(B,S) - G(B,S)^{-j}), \quad \forall j, \tag{A.11}$$

where q^*_{ij} is the optimal solution of $P^V(B,S)$.

A.2. Proof of Theorem 1

First, note that the threshold price p^m used in the transfer rule is determined entirely independently of the bids and asks of the remaining agents. Buyers whose total bids are less than or equal to p^m and sellers with total asks greater than or equal to p^m are systematically excluded. Moreover, within our transfer rule, prices for buyers are higher than or equal to p^m, while sellers' prices are lower than or equal to p^m.

Suppose that a buyer i, with a true valuation b_i, submits a bid $(\hat{b}_i, x_i)$ while assuming other agents bid truthfully. Let $\hat{b}_{ij}$ represent the total bid of buyer i, denoted as $\hat{b}_{ij} = (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij}$. (A.12) illustrates that $\hat{b}_{ij}$ can be greater than p^m for certain sellers or lower than or equal to p^m for others.

$$B^+_i = \{\hat{b}_{ij} | \hat{b}_{ij} > p^m, \quad j \in S\}, \quad \text{and} \quad B^-_i = \{\hat{b}_{ij} | \hat{b}_{ij} \le p^m, \quad j \in S\} \tag{A.12}$$

When buyer i misreports the valuation, two cases arise: (1) $\hat{b}_i > b_i$, and (2) $\hat{b}_i < b_i$. For each case, we demonstrate that there is no incentive for buyers to deviate from truth-telling.

Case 1. $\hat{b}_i > b_i$.

If buyer i can trade with a seller j as a result of $P(B_R, S_R)$, where $\hat{b}_{ij} > p^m$, the trading price is determined by the incentive-compatible modified VCG transfer rule. Consequently, buyer i will not misreport the true valuation.

In cases where the transaction between buyer i and seller j' becomes unfeasible because $\hat{b}_{ij'} \le p^m$, buyer i may overbid to make $\hat{b}_{ij'} > p^m$ and trade with seller j' at a price equal to or higher than p^m. Then, the buyer's utility could turn negative. Additionally, since $\hat{b}_i$ also makes other total bids higher, buyer i cannot obtain better utility from trading with seller j having $\hat{b}_{ij} > p^m$ within our transfer rule. Therefore, buyer i lacks the incentive to misreport and would achieve either the same or lower utility through overbidding.

Case 2. $\hat{b}_i \le b_i$.

If buyer i chooses to underbid, $\hat{b}_{ij}$, initially part of B^+_i, may transition into an element of B^-_i. Even when trading with the sellers corresponding to B^+_i, the buyer cannot attain higher utility due to the constraints imposed by the modified VCG transfer rule. Therefore, buyer i has no incentive to underbid.

When considering both cases, buyers under our double auction mechanism will truthfully bid their true valuation. Similar arguments can be applied to show that sellers will also bid truthfully. Thus, the MVDA mechanism is DSIC.

Since buyers and sellers will submit bids truthfully in the MVDA mechanism, we can consider that $\hat{b}_i = b_i$ for every buyer $i \in B$ and $\hat{a}_j = a_j$ for every seller $j \in S$. Following the final allocation, the trading price for agents is determined through the modified VCG mechanism. The utility of buyer $i \in \tilde{B}$ and seller $j \in \tilde{S}$ are computed as follows:

$$\begin{aligned} u_i &= \sum_{j\in\tilde{S}} \tilde{q}^*_{ij}(1-l_{ij})b_i - \tilde{p}_i \\ &= V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1) \\ &\quad - V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1)^{-i} \end{aligned} \tag{A.13}$$

$$\begin{aligned} u_j &= \tilde{p}_j - \sum_{i\in\tilde{B}} \tilde{q}^*_{ij}a_j \\ &= V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1) \\ &\quad - V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^m, l_{ij} \ne 1)^{-j} \end{aligned} \tag{A.14}$$

For the trading agents, their presence generates additional value compared to the case where they do not participate. This implies non-negative contributions, resulting in both u_i and u_j being non-negative. Hence, the MVDA mechanism is IR.

Determining the trading price $\tilde{p}_i$ for a buyer i involves replacing all total asks of the remaining sellers with the buyers' threshold price p^m. Similarly, for a seller j, this process entails substituting all total bids of the remaining buyers with the sellers' threshold price p^m. Therefore, the total payment made by trading buyers is consistently greater than or equal to the total revenue earned by trading sellers, thereby establishing BB in the MVDA mechanism.

A.3. Proof of Theorem 2

According to the assumption we made for Theorem 2, we can consider that the power loss rate l_{ij} and transaction cost rate t_{ij}—both following some continuous distribution—are bounded. We demonstrate that the MVDA mechanism is AsE for two cases.

Case 1. $\hat{tb} > \hat{ta}$.

In our mechanism, efficiency loss arises from the removed trading relationships. Since the determination of p^+ and p^- corresponds to the total bid of a specific buyer i_0 and the total ask of a seller j_0, respectively, at most two trading relationships will be eliminated among the efficient solutions of $P(B,S)$. Thus, the difference in trading volumes due to the removed trading relationships can be expressed as follows. In (A.15), $\tilde{q}^*_{ij}$ represents the solution of $P(B_R, S_R)$.

$$\sum_{i\in B}\sum_{j\in S} q^*_{ij} - \sum_{i\in\tilde{B}}\sum_{j\in\tilde{S}} \tilde{q}^*_{ij} \le x_{i_0} + y_{j_0} \tag{A.15}$$

Since both x_{i_0} and y_{j_0} are bounded, the left-hand side is also bounded. Moreover, the total bid and the total ask are bounded as well. Hence, the efficiency loss, denoted by $max\{V(B,S)\} - max\{V(B_R,S_R)\}$, remains bounded. As the number of agents approaches infinity, the maximum social welfare $max\{V(B,S)\}$ tends to infinity. Consequently, the efficiency of our double auction mechanism also tends to infinity, as shown in the following equation.

$$\lim_{|B|\to\infty, |S|\to\infty} \frac{max\{V(B_R,S_R)\}}{max\{V(B,S)\}} = 100\% \tag{A.16}$$

Therefore, in Case 1, the MVDA mechanism is asymptotically efficient.

Case 2. $\hat{tb} \le \hat{ta}$.

In this instance, we cannot guarantee the removal of at most two trading relationships among the efficient solutions of $P(B,S)$. Nevertheless, considering our assumption that all agents are situated within a compact domain enabling trade among them, we can identify a finite ε_l-partition $A_1, A_2, \cdot, A_k$ of the compact domain H. Here, ε_l is a positive value, and A_l is a partition with a radius less than ε_l for $l = 1, \ldots, k$. As the number of agents approaches infinity, even with transitions constrained to occur within each partition, the trading volume in each partition under the mechanism will be sufficiently large. We can set the radius ε_l to satisfy the condition:

$$inf\{(1-\alpha\varepsilon_l)b_i - \frac{1}{2}\beta\varepsilon_l | \sum_{j\in S_{A_l}} q^*_{ij} > 0, i \in B_{A_l}\}$$
$$\geq \quad sup\{a_j - \frac{1}{2}\beta\varepsilon_l | \sum_{i\in B_{A_l}} q^*_{ij} > 0, j \in S_{A_l}\} \tag{A.17}$$

We assumed that l_{ij} and t_{ij} are proportional to the distance between a buyer i and a seller j, and the distance is a metric based on the location of the buyer and seller. Hence α and β represent some loss-distance and cost-distance coefficients [34]. Then, within each partition, we can demonstrate that the proposed mechanism is asymptotically efficient using similar arguments as in case 1. Therefore, the MVDA mechanism is also asymptotically efficient in case 2.

Appendix B

B.1. Modified MMTR-MVCG mechanism

We present a modified MMTR-MVCG mechanism based on the double auction model introduced in [34]. To apply their approach to the P2P energy trading problem, we modify the auction mechanism to account for power losses during electricity trading and the transaction costs that buyers and sellers bear together. The modified mechanism also satisfies the required properties in mechanism design.

To adapt the MMTR-MVCG mechanism for the P2P trading context, we set the buyers' threshold price p^+ to $\max\{t\hat{b}, t\hat{a}\}$ and the sellers' threshold price p^- to $\min\{t\hat{b}, t\hat{a}\}$. These p^+ and p^- serve as the criteria for eliminating trading relationships for buyers and sellers, respectively, and as shown in (B.1) and (B.2), they are also used in the VCG transfer rule. All other processes remain similar. The transaction prices for a trading buyer i and seller j are calculated as follows, with all notations carrying the same meanings as those used in Section 3.

$$\begin{aligned}\tilde{p}_i = {} & V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^+, l_{ij} \neq 1)^{-i} \\ & - \{V(B_R, S_R | \hat{a}_j + \frac{1}{2}t_{ij} = p^+, l_{ij} \neq 1) - \sum_{j\in S_R} \tilde{q}^*_{ij}((1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij})\} \\ & + \sum_{j\in S_R} \frac{1}{2}\tilde{q}^*_{ij} t_{ij}\end{aligned} \tag{B.1}$$

$$\begin{aligned}\tilde{p}_j = {} & -V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^-, l_{ij} \neq 1)^{-j} \\ & + \{V(B_R, S_R | (1-l_{ij})\hat{b}_i - \frac{1}{2}t_{ij} = p^-, l_{ij} \neq 1) + \sum_{i\in B_R} \tilde{q}^*_{ij}(\hat{a}_j + \frac{1}{2}t_{ij})\} \\ & - \sum_{i\in B_R} \frac{1}{2}\tilde{q}^*_{ij} t_{ij}\end{aligned} \tag{B.2}$$

B.2. Proof of Proposition 1

For trading agents under the MVDA mechanism, two cases arise: (1) they also engage in trading under the modified MMTR-MVCG mechanism, or (2) they are unable to trade under the modified MMTR-MVCG mechanism. In case 1, the following conditions hold:

$$\tilde{p}^{MO}_i \geq \sum_{j\in \tilde{S}^{MO}} \tilde{q}^{*,MO}_{ij}(p^+ + \frac{1}{2}t_{ij}) \tag{B.3}$$

$$\tilde{p}^{MV}_i \geq \sum_{j\in \tilde{S}^{MV}} \tilde{q}^{*,MV}_{ij}(p^m + \frac{1}{2}t_{ij}) \tag{B.4}$$

Here, "MO" in the proof denotes the values determined by the modified MMTR-MVCG mechanism, while "MV" refers to the values determined by the MVDA mechanism. The buyers' and sellers' sets, along with their bid information, remain consistent across both mechanisms. In addition, the set of remaining buyers (sellers) under the modified MMTR-MVCG mechanism is a subset of the remaining buyers (sellers) under our MVDA mechanism. Therefore,

$$inf\{(1-l_{ij})b_i - \frac{1}{2}t_{ij} | i \in B^{MO}_R, j \in S^{MO}_R\}$$
$$\geq \quad inf\{(1-l_{ij})b_i - \frac{1}{2}t_{ij} | i \in B^{MV}_R, j \in S^{MV}_R\}. \tag{B.5}$$

Thus, $\tilde{p}^{MO}_i \geq \tilde{p}^{MV}_i$. Since the allocation is based on the maximum social welfare among the remaining agents, buyer i will trade an equal or greater volume than under the modified MMTR-MVCG mechanism. Therefore, buyers in case 1 obtain the same or greater utility under the MVDA mechanism. This argument extends to sellers for similar reasons.

By Theorem 1, the MVDA mechanism is individually rational, thereby offering equivalent or enhanced utility to agents in case 2. For agents unable to trade under the MVDA mechanism, their inability to trade is also observed under the modified MMTR-MVCG mechanism, resulting in zero utility. Therefore, Proposition 1 holds for all agents, leading to improved social welfare.

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