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IS IMITATION CONDUCIVE TO COOPERATION IN LOCAL INTERACTION MODELS?*

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We ask whether imitation is conducive to cooperation in local interaction models. We show that the prediction depends significantly on the specific imitation rules and interaction structures.

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Ⅰ. INTRODUCTION

In a series of papers, Eshel and his co-authors (Eshel et al., 1998; Eshel et al., 1999; Eshel et al., 2000) recently established that imitation and local interaction can sustain cooperation. They assumed that (i) individuals interact only with their small neighbors, and that (ii) they update their behavior by imitating the behavior of their successful neighbors. With these plausible assumptions, they were able to show that cooperative behavior or altruism is possible. In particular, they have argued that, if cooperation survives at all, it will be present in large numbers. In a representative model in Eshel, Samuelson and Shaked (1998, ESS henceforth),[1] it is shown that the average proportion of Altruists, that is, individuals who adopt cooperative behavior, is at least 60% of the population in any absorbing set in which at least one Altruist exists.

This paper is intended to show that this result is not robust. We change a

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[1] The reader should be aware that ESS here does not represent the Evolutionarily Stable Strategies.

single assumption in ESS's paper slightly and establish the opposite result that the proportion of Altruists in any absorbing set is generally much smaller than the initial probability of Altruists. The assumption we change is the specific rule for imitation.

ESS assume that each individual, when given an opportunity to learn, observes the behavior of his neighbors, calculates the average payoff of each behavior (cooperative behavior and defective behavior), and adopts the behavior that awards the higher payoff. In this paper, we assume instead that each individual adopts the behavior of the individual in his neighborhood who achieves the highest payoff. That is, instead of the average criterion in ESS, we take the maximum criterion.

Which criterion is better or more plausible? The answer probably depends on specific situations. In many cases, people try to imitate the individual who achieves the most: 'We imitate whom we adore,' as St. Augustine put it. In other cases, especially with risk aversion, people may choose the alternative that produces a better payoff on average than the alternative that produces the best payoff for a special occasion. As for practical matters such as economic and social development, the main point of the present paper, which shows the maximum criterion produces less cooperation suggests some policy intervention or social initiative towards the adoption of the average criterion would be desirable for the society as a whole.

In the next section, we study the imitation dynamics of the maximum criterion and compare this with that of the average criterion. The driving force behind the result that the final proportion of Altruists under the maximum criterion is much smaller than that under the average criterion is the fact that the former criterion does not allow an Egoist to imitate an Altruist while the reverse is not true. In Section Ⅲ, we consider larger neighborhoods and find that the sharp difference between the two criteria is somewhat mitigated. This section shows that the maximum criterion may allow an Egoist to imitate an Altruist when the neighborhoods become larger. We rely on simulations for some of the results. The upshot of the analysis is that the specifics matter for imitation dynamics in local interaction models. Section Ⅳ concludes.

Ⅱ. MAIN RESULTS

A. The Model

The model is the same as that in ESS. Assume a finite population of individuals located around a circle. Each individual interacts only with his immediate neighbors (the individual to his left and the individual to his right) in each period $t=1,2,\cdots$. Individuals decide whether to behave cooperatively or not. Or, to follow ESS, individuals decide whether to be an Altruist or an Egoist. An Altruist provides a public good to his neighbors with a net cost of $C>0$. The benefit from the public good is normalized to 1 and the cost C is assumed to be less than 1. An Egoist provides no public good and only enjoys the benefits provided by others.

The imitation dynamics in this paper is as follows. At the end of each period, each individual observes his neighbors' behavior and payoffs as well as his own, and switches to the behavior that has produced the highest payoff. In other words, each individual imitates the most successful individual in his neighborhood (consisting of his two neighbors and himself) in every period. For example, if an Altruist enjoys the highest payoff among his two neighbors and himself, then he chooses to be an Altruist in the next period.[2]

In contrast, each individual in ESS is assumed to switch to the behavior whose average payoff is higher. Hence, each individual calculates the average payoffs of Altruists and of Egoists in his neighborhood, and then chooses to be an Altruist if the average payoff of Altruists exceeds that of Egoists and vice versa. We call the latter specification as *the average criterion*, while the former as the *maximum criterion.*

B. Equilibrium

We now characterize *absorbing sets*. Recall that an absorbing set is a set of states with the property that the Markov process can lead into this set but not out of it. Since each individual either retains his own behavior or adopts a behavior played by one of the two individuals in his neighborhood, and the behaviors as well as the payoffs of the latter two individuals depend only on the behaviors of the next two neighbors, the imitation dynamics of each individual is completely determined by the behaviors of his four nearest

[2] Please refer to ESS for a justification of imitation dynamics.

neighbors.

We first investigate whether an Egoist may turn into an Altruist. When an Egoist is surrounded by two Egoists as displayed below,[3]

$$xE \quad E \quad Ex$$

he will remain an Egoist since he observes only Egoists in his neighborhood. When an Egoist is surrounded by two Altruists,

$$xa \quad E \quad ax$$

the Egoist in the center obtains the highest payoff of 2 (while the Altruist neighbors obtain at most $1-C$) and so will remain an Egoist regardless of x's next to the Altruists. As for the remaining case

$$xE \quad E \quad ax$$

and its mirror image, we can also see that the Egoist in the center will remain an Egoist since the Egoist obtains at least 1 while the Altruist neighbor obtains at most $1-C$. Therefore, an Egoist will always remain an Egoist under the maximum criterion.

An Altruist will remain an Altruist if

$$xa \quad a \quad ax$$

or

$$EE \quad a \quad aa$$

or the mirror image of the second configuration since he observes only Altruists or his Altruist neighbor on the right obtains the maximum payoff of $2-C$. In all other cases, he will become an Egoist.

From the above discussion, we obtain the following result.[4]

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[3] Following ESS, we use a lower case " a " to represent an Altruist, an upper " E " to represent an Egoist, and a lower " x " to represent any type of individual.

[4] A string is a maximum interval of individuals of the same type.

Proposition 1. *For the maximum criterion imitation dynamics, the absorbing sets are (i) the state in which all are Altruists, (ii) the state in which all are Egoists, and (iii) states in which strings of at least 3 Altruists are separated by strings of at least 2 Egoists.*

Note that every absorbing set consists of a single steady state. We now compare this result with ESS's. ESS obtain, under the average criterion and the additional assumption of $C<1/2$, that (a) there are cases when an Egoist becomes an Altruist, and moreover (b) the average proportion of Altruists in any absorbing set, except for the state in which all individuals are Egoists, is at least 0.6. What we establish here is that this observation is not robust since it can be destroyed with a single change of the average criterion to the maximum criterion, retaining all the other structures in ESS. In fact, the average proportion of Altruists is much smaller than the ex-ante probability of Altruists. We now address the dynamic process to verify this assertion.

Suppose that a string of Altruists is *persistent* if it does not disappear during the dynamic process. Then, we have,[5]

Proposition 2. *A string of Altruists is persistent if and only if (i) the string contains at least five Altruists, (ii) the string consists of four Altruists bordered on at least one end by two Egoists, or (iii) the string consists of three Altruists bordered on each end by two Egoists.*

From the proposition, we observe that the system is at a steady state by period 3. From the initial condition, strings of Altruists that are not persistent will disappear while all strings of Egoists will remain or expand. After 2 transition periods, the system will reside in one of the steady states. What is the average proportion of Altruists at steady states when we begin with an arbitrary initial condition?

To give an answer to this, now suppose that individuals' identities in the initial condition be randomly and independently determined, with probability p

[5] One of the referees provided the following very helpful point: While a string containing at least five Altruists "persists" in Eshel et al. in a sense that *E aaaaa E* becomes *E EaaaE E* and then back to *E aaaaa E*, the same is not true with the maximum criterion because *E aaaaa E* becomes *E EaaaE E* and remains the same at *E EaaaE E*. So, while the whole string does not disappear, a string containing at least five Altruists reduces to a string containing only three Altruists under the maximum criterion.

attached to being an Altruist. We report the simulation results for the maximum criterion as well as the average criterion for comparison. In Table 1, the column MAXIMUM (AVERAGE, respectively) shows the final proportion of Altruists for the maximum (average, respectively) criterion for different values of p.

[Table 1] Two neighbors

p	MAXIMUM	AVERAGE
0.1	0.002	0.040
0.2	0.015	0.245
0.3	0.040	0.538
0.4	0.081	0.754
0.5	0.141	0.845
0.6	0.225	0.866
0.7	0.344	0.864
0.8	0.506	0.870
0.9	0.724	0.912

Note. The column MAXIMUM (AVERAGE, respectively) shows the final proportion of Altruists for the maximum (average, respectively) criterion for different values of p. We set the number of individuals to 20 for the maximum criterion, which is sufficiently large given the above propositions. That is, we will obtain the same results for larger populations. On the other hand, we set the number of individuals to 50 for the average criterion to represent a large population. Recall from ESS that we will obtain higher final proportions for the average criterion if we increase the population. In both cases, we set the net cost C to 0.4. Note that any C strictly between 0 and 1 will give the same results for the maximum criterion, while only C's strictly between 0 and 1/2 will give the same results for the average criterion. The simulation was run for 200,000 times for the maximum criterion, and for 20,000 times for the average criterion.

As can be seen from the above table, the final proportion of Altruists under the maximum criterion is much smaller than probability p. This result holds uniformly for any reasonably large population. This is in nice contrast with the average criterion under which the final proportion exceeds the initial probability. The contrast is due to the fact that Egoists never switch behavior while strings of Altruists that are not persistent disappear under the maximum criterion, while persistent strings of Altruists will expand until the strings of Egoists contract to size 2 under the average criterion.

Recall that the final proportion of Altruists under the average criterion is at least 60% in any absorbing set in which at least one Altruist exists, regardless of ex-ante probability p. Since the simulation results include the possible initial conditions which render Altruists eventually disappear, the reported proportions are lower for low p's. We finally note that, since there may exist cycles and so the system may not settle down to a steady state under the average criterion, we stop the dynamics after 50 periods and investigate the configurations at that time. We expect that this will capture the average configuration quite correctly with enough simulations. On the contrary, there does not exist cycles but the system settles down after 3 periods under the maximum criterion.

Ⅲ. LARGER NEIGHBORHOODS

The stylized model in the previous section raises the question, 'What is a correct description of the imitation dynamics in local interaction models?' In this section, we consider some variants of local interaction structure to delineate the general picture.

Let us now assume that each individual interacts with two individuals to his left and two individuals to his right. That is, now the neighborhoods are of radius two rather than one. At first glance, it appears that there will remain less Altruists in this environment. Consider the extreme case when each individual interacts with the whole population. Then, Altruists hold out no hope since the cooperative behavior is strictly dominated by the defecting behavior. It turns out, however, that this impression is wrong.

It is now the eight nearest neighbors who determine the imitation dynamics of each individual. The analysis is more challenging, but the calculation reveals the important fact that there are neighborhood configurations which turn an Egoist into an Altruist even under the maximum criterion. These configurations are

$$xxEE\ E\ aaaa$$

and its mirror image. There are also many possible configurations that keep an Altruist as an Altruist. The dynamic process consequently exhibits a complicated pattern, which makes analytical investigation impractical.[6] We, therefore, resort to simulation and report the following result. We again report the simulation results for the maximum criterion as well as the average criterion for comparison. As in the previous section, individuals' identities in the initial condition are randomly and independently determined, with probability p attached to being an

[6] There are cycles as well as single steady states among the absorbing sets. Consequently, in the simulation below, we stop the dynamics after 50 periods and investigate the configurations at that time.

Altruist.

[Table 2] Four neighbors

p	MAXIMUM	AVERAGE1	AVERAGE2
0.1	0.002	0.013	0.001
0.2	0.025	0.092	0.008
0.3	0.091	0.239	0.042
0.4	0.228	0.400	0.107
0.5	0.420	0.536	0.194
0.6	0.622	0.645	0.320
0.7	0.777	0.776	0.538
0.8	0.852	0.868	0.788
0.9	0.878	0.914	0.898

Note. The column MAXIMUM shows the final proportion of Altruists for the maximum criterion, while AVERAGE1 (AVEREAGE2, respectively) shows the final proportions of Altruists for the average criterion when the net cost $C=0.4$ ($C=0.9$, respectively). We set the number of individuals to 40 for the maximum criterion, and to 50 for the average criterion. The simulation was run for 100,000 times for the maximum criterion, and for 20,000 times for the average criterion.

In Table 2, the column MAXIMUM shows the final proportion of Altruists for the maximum criterion, while AVERAGE1 (AVEREAGE2, respectively) shows the final pro- portions of Altruists for the average criterion when the net cost $C=0.4$ ($C=0.9$, respectively).[7]

There exist more Altruists under the maximum criterion compared to the two neighbors case, but they are still not dominant. On the other hand, there exist *fewer* Altruists under the average criterion. Hence, the sharp difference found in the two neighbors case is mitigated. The main reason for this convergence is that complex imitation patterns emerge, especially the pattern in which an Egoist turns into an Altruist, when people interact in a larger neighborhood. We can also confirm our intuition from AVERAGE1 and AVERAGE2 that the higher costs of altruism reduce the proportion of Altruists.[8]

[7] As before, the exact level of C is not important for the maximum criterion as long as it is strictly between 0 and 1.

[8] ESS examine larger neighborhoods and make the following point: decreasing the cost of altruism can be bad for Altruist. The current result shows that their assertion is not general.

IV. CONCLUSION

We have found that both Altruists and Egoists coexist under the imitation dynamics on the local interaction structure. The extensive simulation results of Nowak and others for two-dimensional neighborhoods, nicely summarized in Nowak et al. (1994), also confirm this observation.

It is perhaps not surprising to observe the cooperative behavior or altruism, which is a strictly dominated behavior, under the imitation dynamics. While it is certainly true that people use some imitative component when revising their behavior, imitation alone is a pretty naive learning rule. On top of the intrinsic weakness of imitation as a plausible explanatory variable of human behavior, we find in this paper that it matters significantly for the imitation dynamics how we specify the imitation rules and the interaction structures.[9] It is worthwhile to find out which properties of learning rules and interaction structures determine the proportion of cooperative behavior.

[9] Hence, we emphasize the sensitivity of imitation dynamics to model specifications. Other works, including Bergin and Lipman (1996) and Robson and Vega-Redondo (1996), emphasized the sensitivity of best-response dynamics to model specifications.

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